Requiring Minimum Sales Volume to Trigger a Commission Increase

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Outline

Why sophisticated contracts?

- Simple contracts make the chain uncoordinated:
  - Double marginalization
  - Low stocking
  - Ordering cost

Why sophisticated contracts?

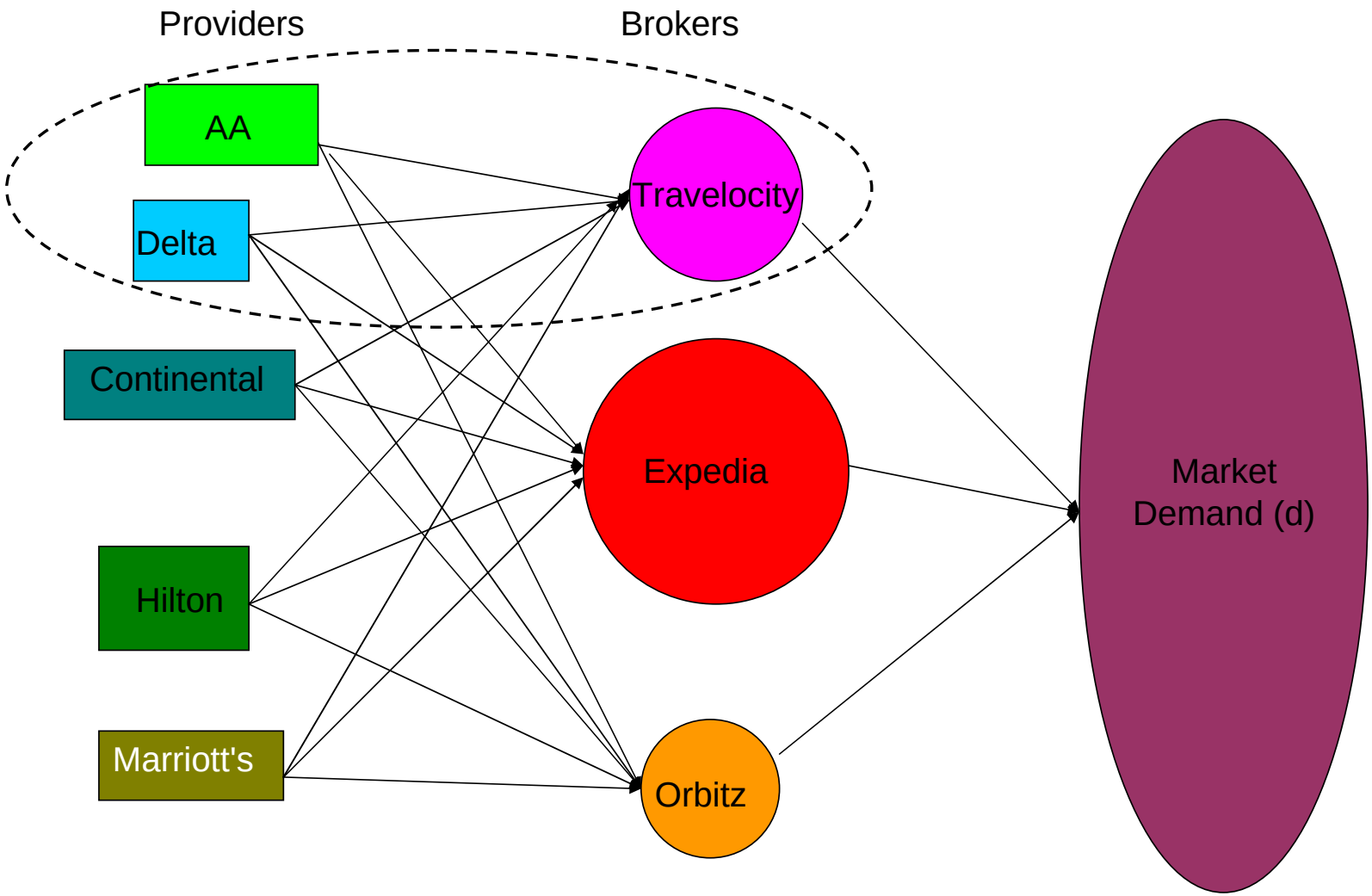
- Simple contracts make the chain uncoordinated:
  - Double marginalization
  - Low stocking
  - Ordering cost
- Sophisticated contracts to achieve coordination:
  - Quantity discount: Weng (1995)
  - Sale rebate (target rebate): Gallego et al. (2008)
  - Full return (buy-back): Tsay and Lovejoy (1999)
  - Revenue Sharing: Cachon and Lariviere (2005)

Sale contracts and Commission contracts

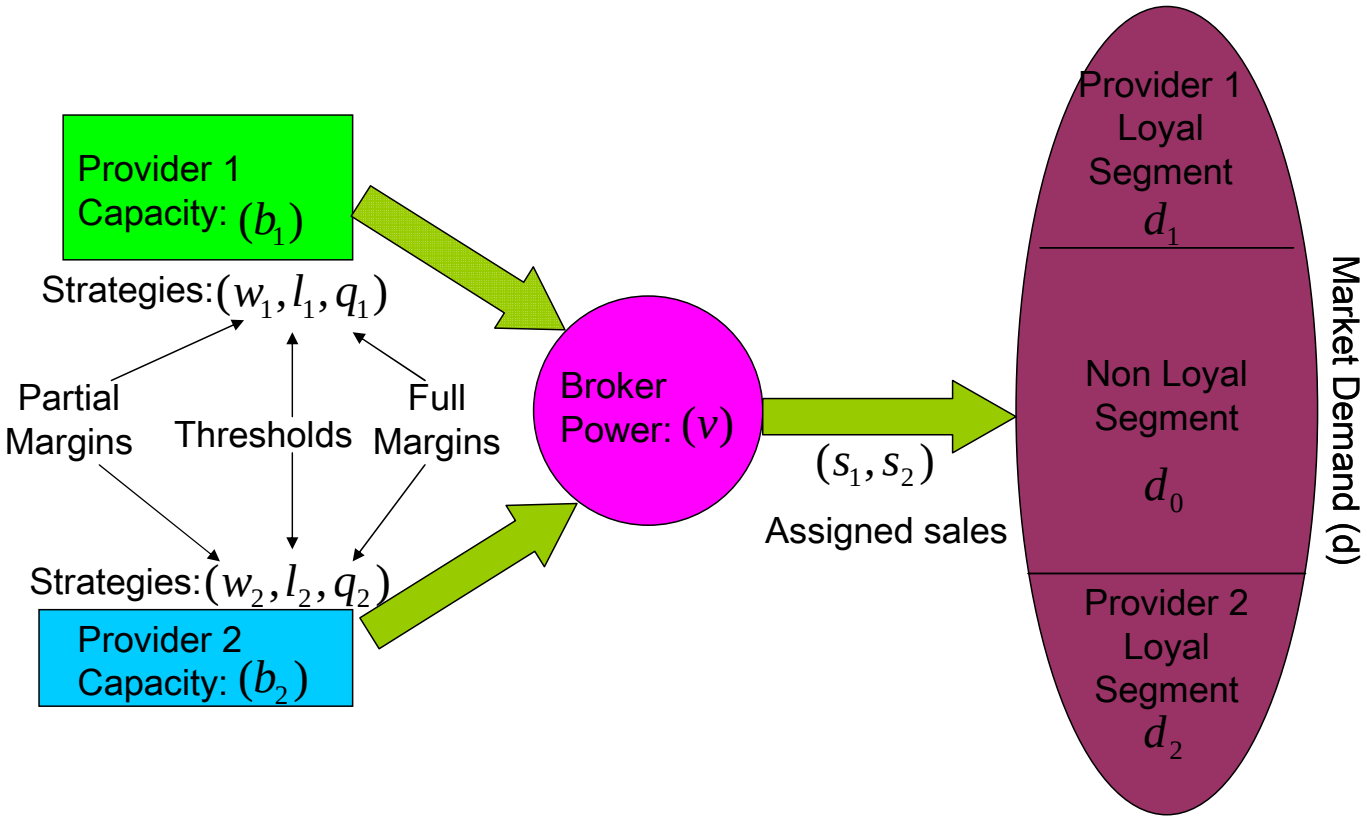
- Retailer buys the capacity from the supplier.
  - Supplier requires minimum sale volume to trigger quantity discounts.

- Provider pays broker a commission margin on each sale.
  - Provider requires minimum sale volume to trigger a commission increase.

Service Industry: Players Selection



Problem definition



We assume that the sales price of products is exogenous and fixed at *p*.

Modelling demand

Assumptions:

- As d increases, d_0 and d_is increase proportionally.
- As v increases, d_0 increases and d_is decrease.

These assumptions are satisfied by:

- Multinomial Logit (MNL) Choice: $d_i = \frac{e^{(u_i - p)}}{e^{(u_i - p)} + e^{(u_j - p + \nu(v))}} d$
- Market Segmentation: $d_i = \beta_i(1 - \alpha(v))d$

Outline

Formulation

Broker Problem:

$$\max_{(s_i,\theta_i)} \quad \pi_B = q_1 s_1 + q_2 s_2$$

$$0 \leq s_i \leq \min\{b_i, d_i + \theta_i d_0\} \qquad \text{for } i = 1,2$$

$$\theta_1 + \theta_2 = 1$$

$$0 \leq \theta_i \qquad \text{for } i = 1,2$$

Providers' Best Response Problem:

$$\max_{(q_i)} \quad \pi_i(q_{3-i}) = (p - q_i)s_i \qquad \text{for } i = 1,2$$

$$0 \leq q_i \leq p$$

Theorem 1

- Assume $b_i > \max\{d_i, d - b_j\}$ and call it a competitive market.
- Define $m_i = \min\{b_i, d_0 + d_i\}$.
- Label the provider with higher m, provider 1 and the primary.

There exists a mixed-strategy Nash equilibrium such that for $q \in [0, \frac{m_1+m_2-d}{m_1}p]$

$$P(q_1^* \leq q) = \frac{p[m_2(d-m_2) - m_1(d-m_1)] + qm_1(d-m_1)}{(p-q)(m_1+m_2-d)m_1}$$

$$P(q_2^* \leq q) = \frac{q(d-m_2)}{(p-q)(m_1+m_2-d)}$$

Magnitude of commission margins

In equilibrium, the primary provider pays stochastically smaller commission margins.

Competitive market

Market situation	Revenue split	$\frac{\delta \pi_i}{\delta d}$	$\frac{\delta \pi_i}{\delta v}$
$b_1 < d_0 + d_1$	$\pi_1 = p \max[d - b_2, d_1]$	> 0	≤ 0
	$\pi_2 = p \frac{\min[b_2, d_0 + d_2]}{b_1} \max[d\text{-}b_2, d_1]$	> 0	$\in \Re$
	$\pi_B = p(d\text{-}\frac{\min[b_2, d_0 + d_2] + b_1}{b_1} \max[d\text{-}b_2, d_1])$	$\in \Re$	≥ 0
$b_1 > d_0 + d_1$	$\pi_1 = p \max[d - b_2, d_1]$	> 0	≤ 0
	$\pi_2 = p \frac{\min[b_2, d_0 + d_2]}{d_0 + d_1} \max[d\text{-}b_2, d_1]$	> 0	$\in \Re$
	$\pi_B = p(d\text{-}\frac{\min[b_2, d_0 + d_2] + d_0 + d_1}{d_0 + d_1} \max[(d\text{-}b_2), d_1])$	$\in \Re$	> 0

Figure 1: Revenues when demand is not loyal

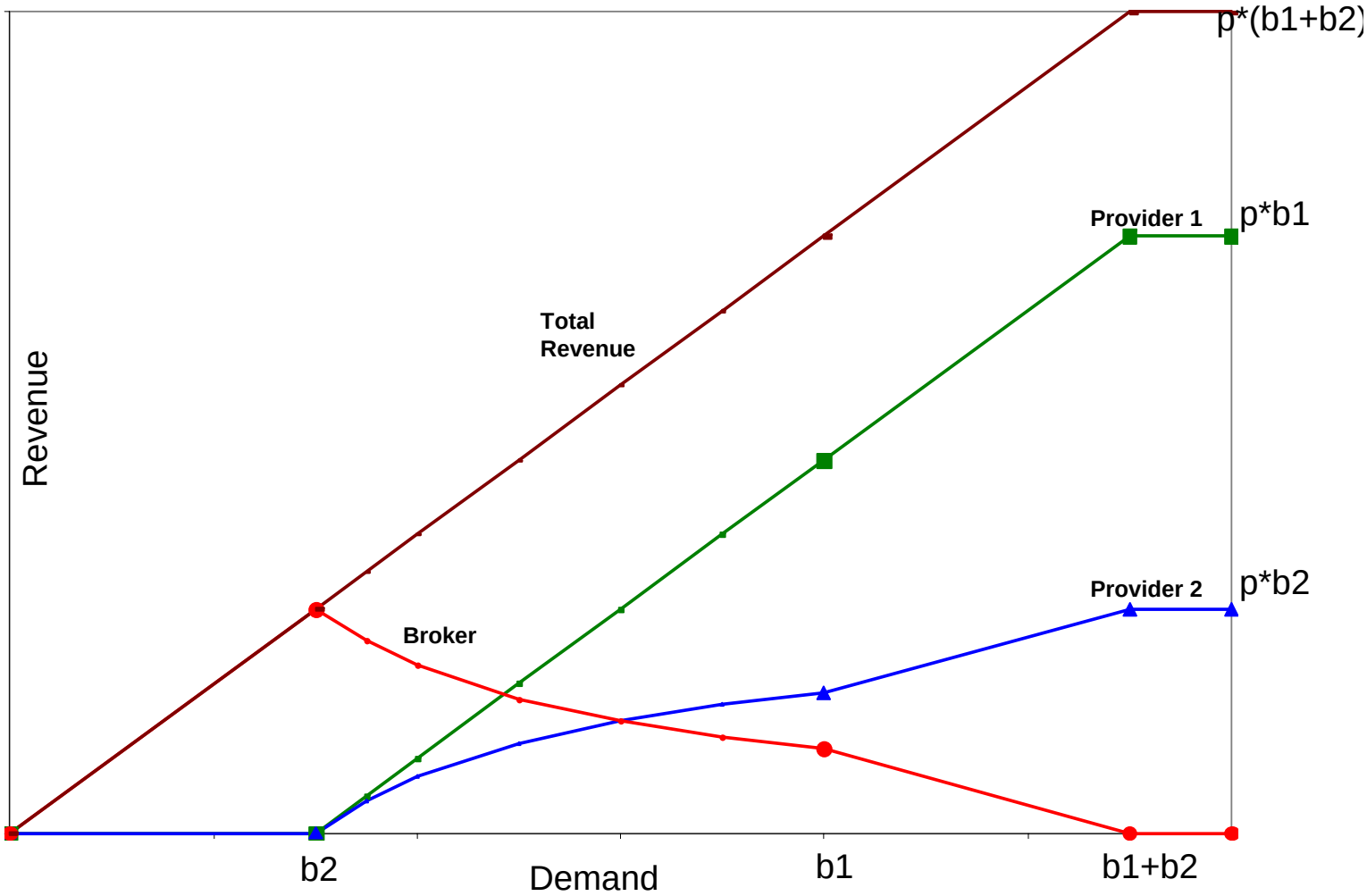
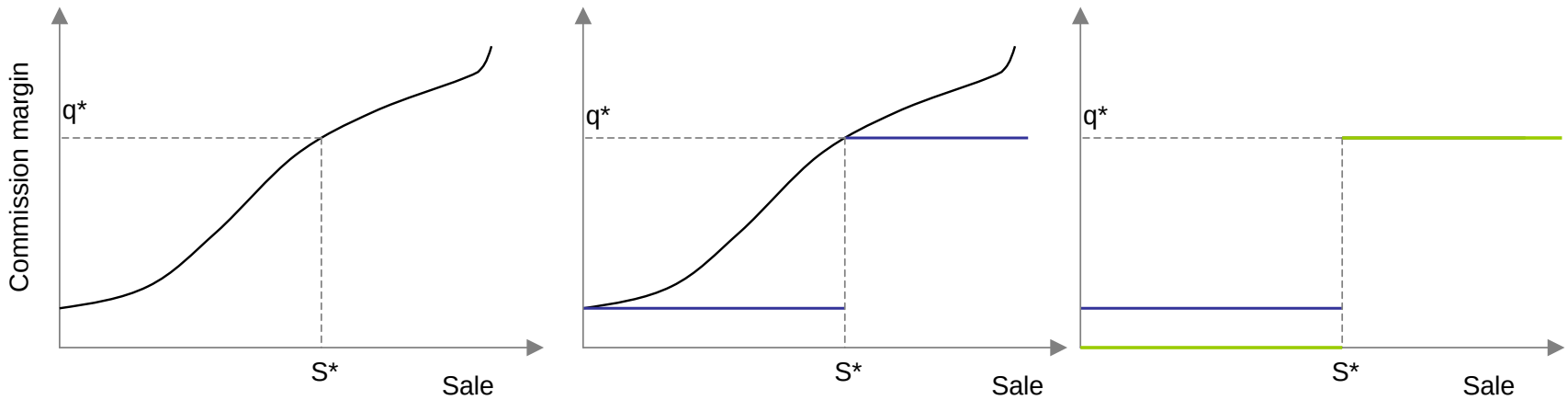


Figure: Flexible margins without thresholds

Assumption of $w = 0$ is w.l.o.g.

Corresponding to any non-negative and nondecreasing commission margin, there exists a simple commission margin function with one breakpoint starting from 0 that results in the same amount of sale and the same commission payment.



Formulation

Broker Problem:

$$\max_{(s_i,\theta_i,k_i,\Delta_i)} \quad \pi_B(l_1,l_2) = k_1 q_1 s_1 + k_2 q_2 s_2 - p(\Delta_1+\Delta_2)$$

$$k_i l_i \le s_i \le \min\{b_i, d_i + \theta_i d_0 + \Delta_i\} \qquad \text{for } i = 1,2$$

$$\theta_1 + \theta_2 = 1$$

$$k_i \in \{0,1\} \qquad \text{for } i = 1,2$$

$$0 \le \Delta_i, \theta_i \qquad \text{for } i = 1,2$$

Δ_i : Purchased units by the broker from provider i, in excess of demand to trigger a commission increase.

Providers' Best Response Problem:

$$\max_{(l_i,q_i)} \quad \pi_i(l_{3-i}, q_{3-i}) = (p - k_i q_i) s_i \qquad \text{for } i = 1,2$$

$$0 \le l_i \le b_i$$

$$0 \le q_i \le p$$

Theorem 2

In a competitive market, there exists a pure-strategy Nash equilibrium such that:

$$q_i^* = (\frac{m_1 + m_2 - d}{m_i})p$$

The equilibrium results in

$$s_i^* = l_i^* = m_i$$

Magnitude of commission margins

In equilibrium, the primary provider pays smaller commission margins.

Competitive market

Market situation	Revenue split	$\frac{\delta \pi_i}{\delta d}$	$\frac{\delta \pi_i}{\delta v}$
$b_1 < d_0 + d_1$	$\pi_1 = p \max[d - b_2, d_1]$	> 0	≤ 0
	$\pi_2 = p(d - b_1)$	> 0	0
	$\pi_B = p(b_1 - \max[d - b_2, d_1])$	< 0	≥ 0
$b_1 > d_0 + d_1$	$\pi_1 = p \max[(d - b_2), d_1]$	> 0	≤ 0
	$\pi_2 = p d_2$	> 0	< 0
	$\pi_B = p(d_0 + d_1 - \max[d - b_2, d_1])$	$\in \Re$	> 0

Figure 2: Revenue when demand is not loyal

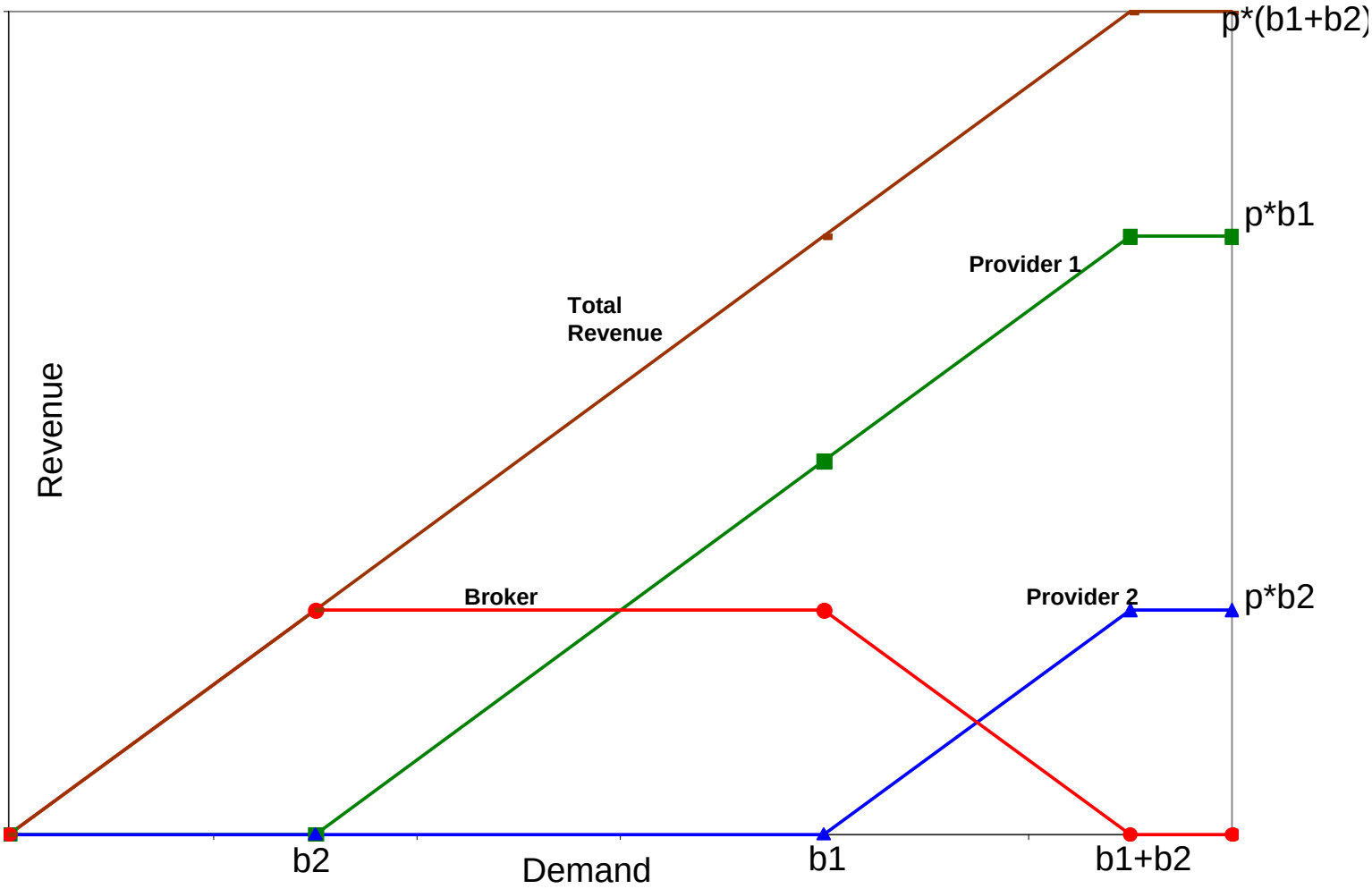


Figure: Flexible margins with thresholds

Figure 3: Revenues when demand is large and loyal

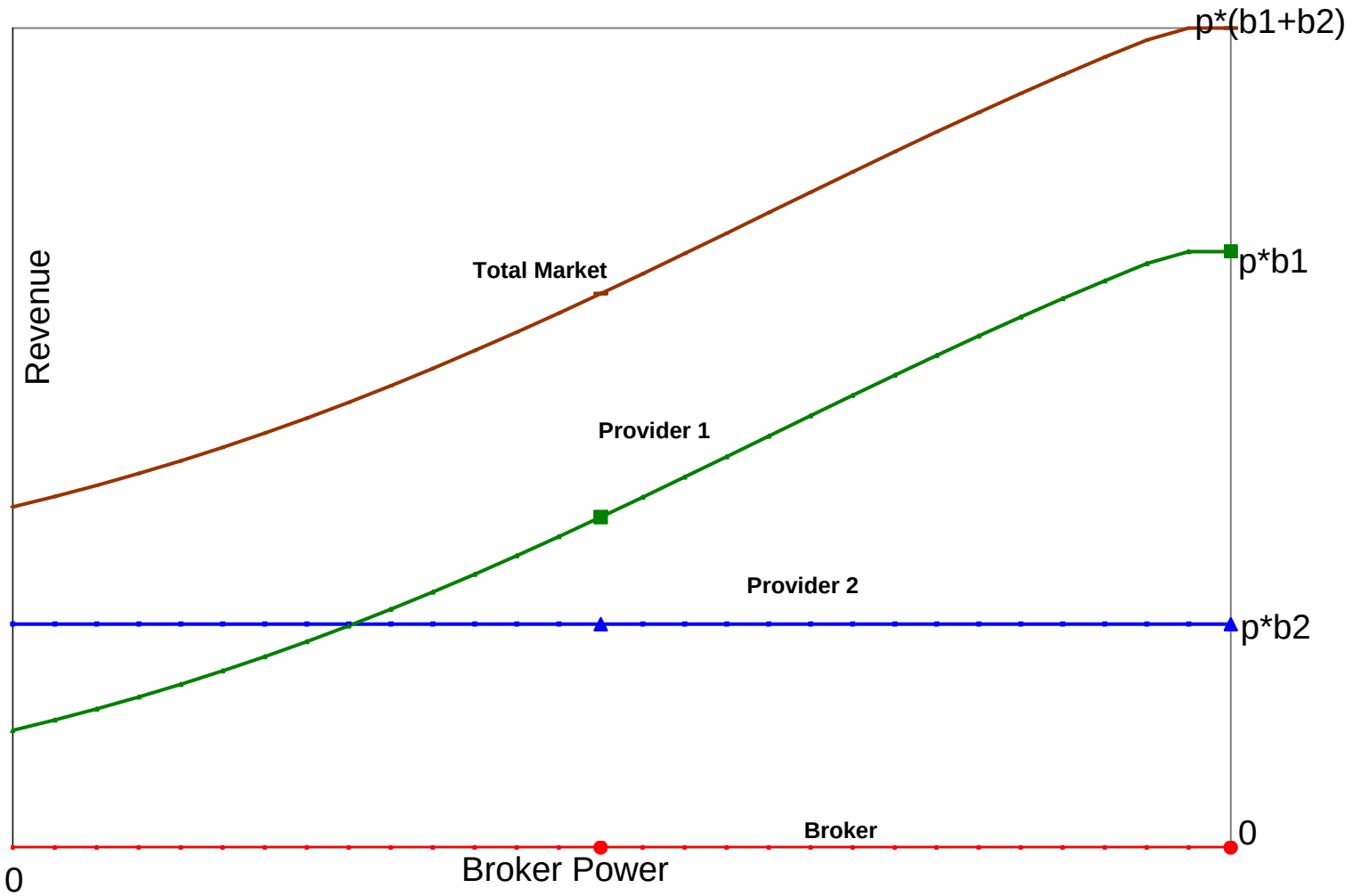


Figure: Flexible margins with or without thresholds

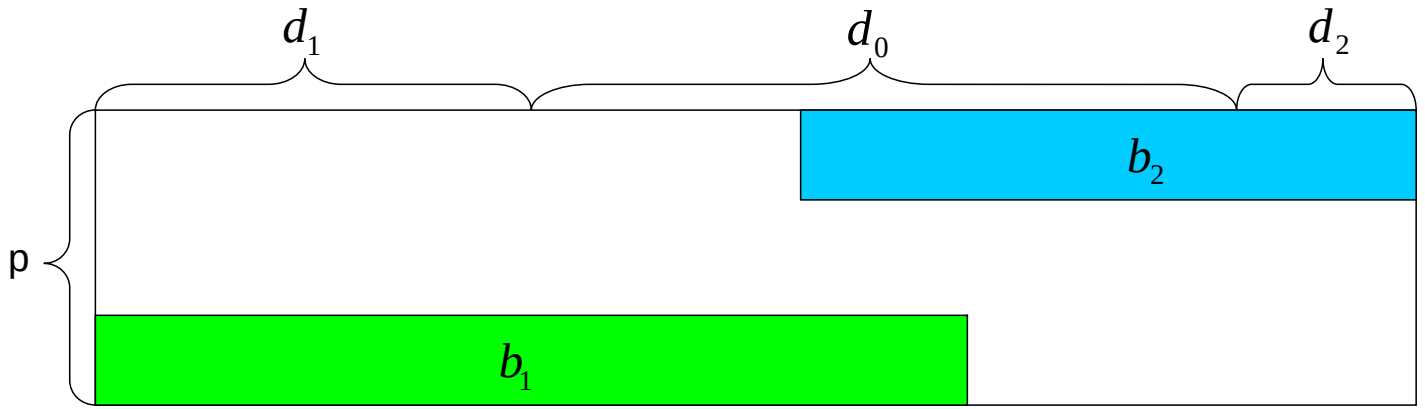
Effect of the market demand and broker power

	Without thresholds		With thresholds	
	$/\delta d$	$/\delta v$	$/\delta d$	$/\delta v$
$\delta\pi_1$	> 0	≤ 0	> 0	≤ 0
$\delta\pi_2$	> 0	$\in \Re$	> 0	≤ 0
$\delta\pi_B$	$\in \Re$	≥ 0	$\in \Re$	≥ 0

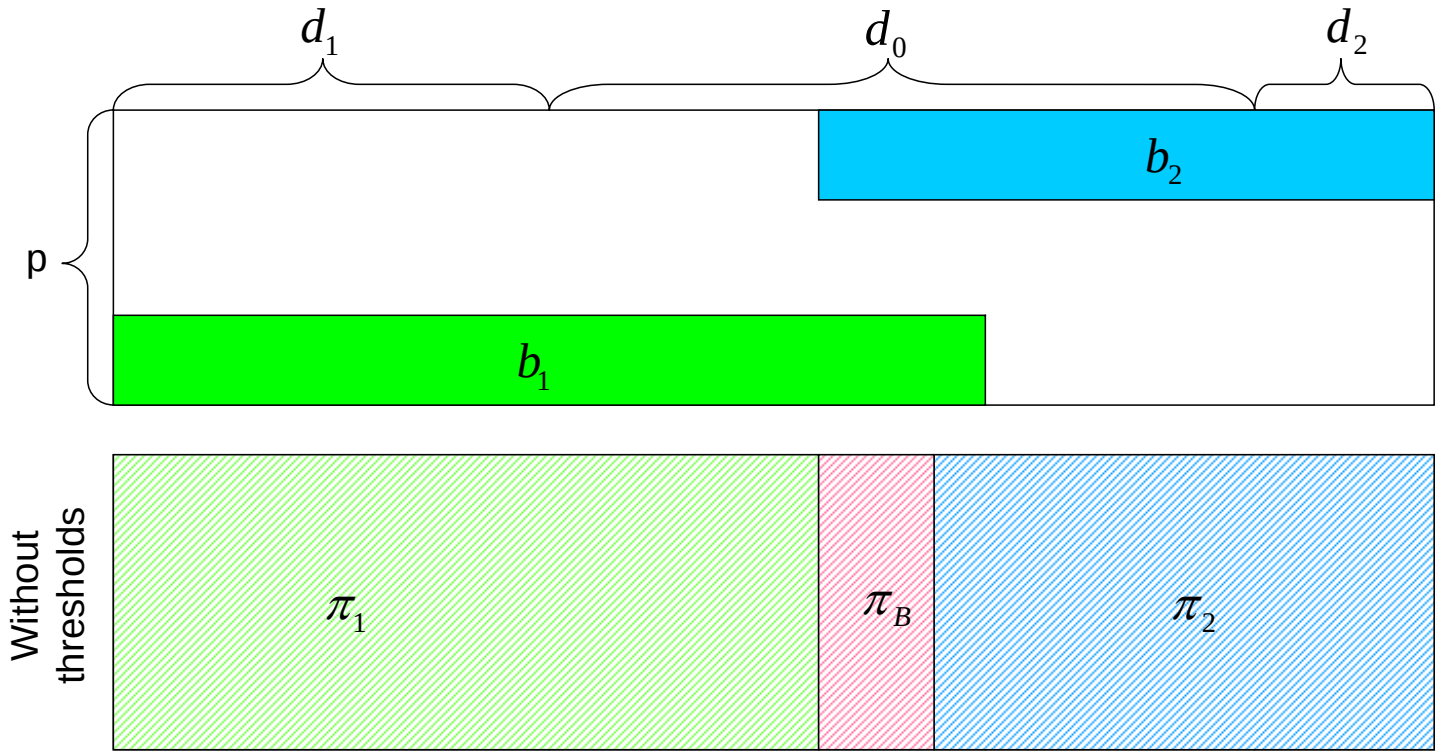
Who loses and who wins in a competitive market

- The primary provider's revenue remains "fixed".
- The secondary broker "loses".
- The broker "wins".

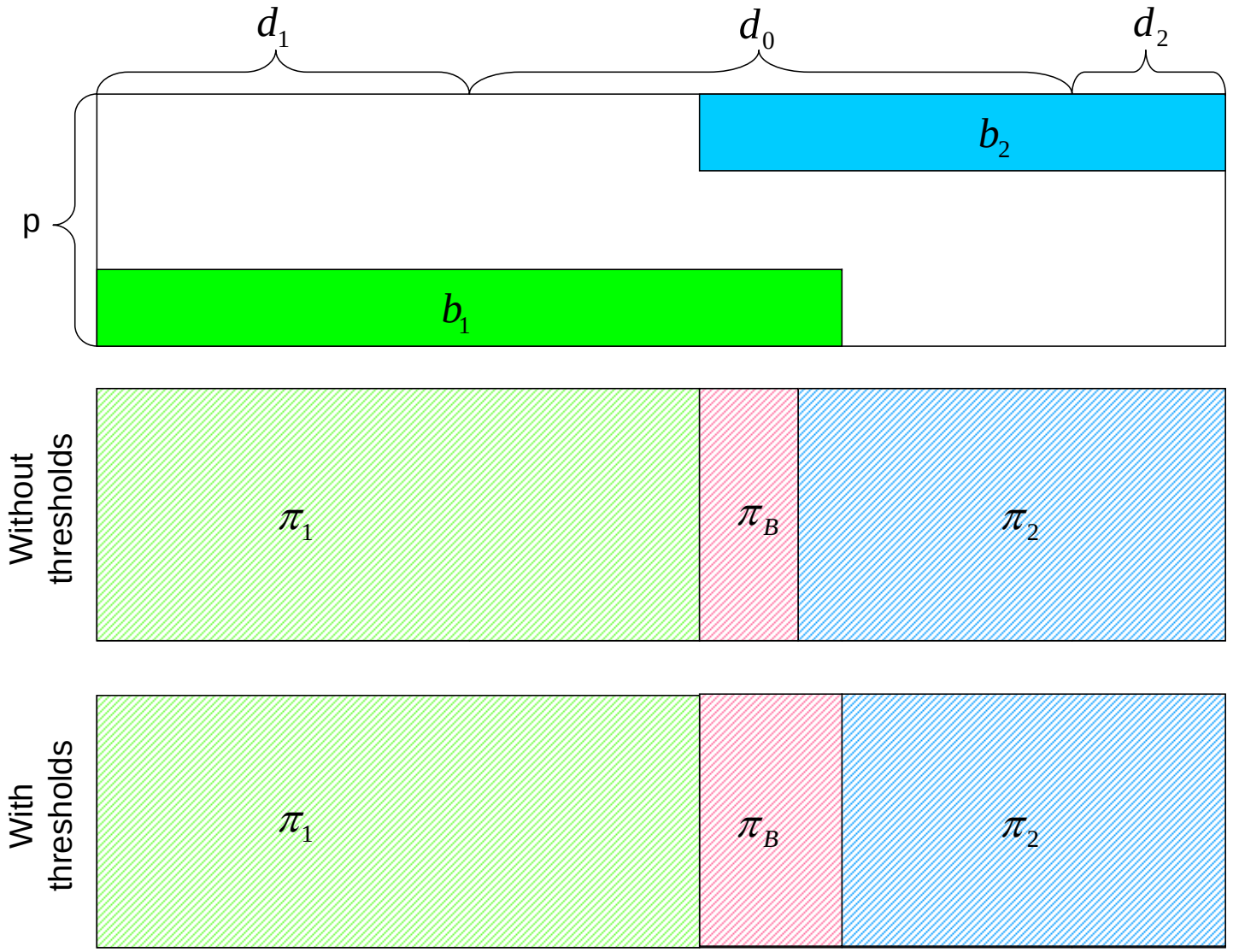
Strategic effects in a competitive market



Strategic effects in a competitive market



Strategic effects in a competitive market



Paradox!

- Providers are not winning by introduction of thresholds.
- Yet, there is a big push by providers to introduce the thresholds.
- Why?!

Outline

Who loses and who wins in a competitive market ($q_1 > q_2$)

- Ay least one of the providers “wins”.

		$b_2 < d_0 + d_2$	$d_0 + d_2 < b_2$
$b_1 < d_0 + d_1$	$\hat{\pi}_1 \geq \hat{\pi}_2$	Fixed , Win	Fixed , Win
	$\hat{\pi}_1 < \hat{\pi}_2$	Loss , Win	Loss , Win
$b_1 > d_0 + d_1$	$\hat{\pi}_1 \geq \hat{\pi}_2$	Win$^+$, Win	Win$^+$,Win
	$\hat{\pi}_1 < \hat{\pi}_2$	Loss , Win	Loss , Win

- The broker “loses”.

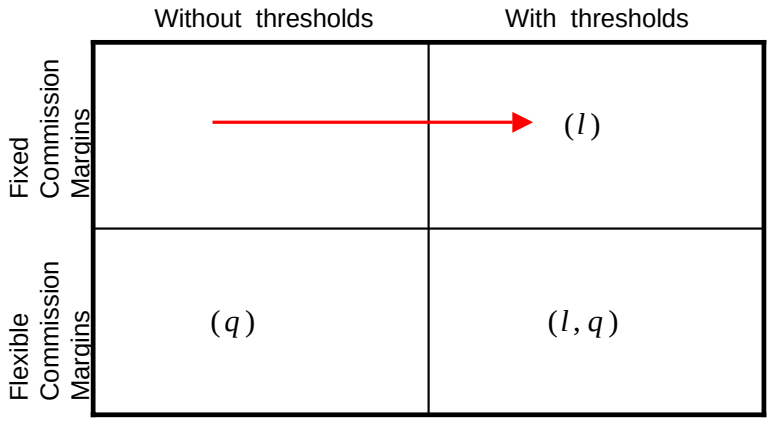
$+$ Win unless $\hat{\pi}_1 = \hat{\pi}_2$

$\hat{s}_i = \max[m_i, \frac{p}{p-q_i}(d - m_j)]$

$\hat{\pi}_i = q_i \hat{s}_i - p(\hat{s}_i - m_i)$

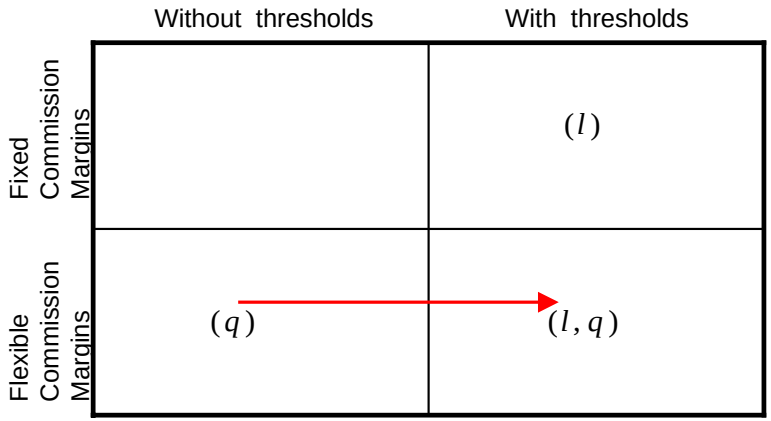
Outline

Conclusions when margins are fixed



- The provider with the higher total commission fee, which in addition to commission margin depends on the available capacity and the loyal market too, gets prioritized.
- The broker loses and the secondary provider wins. The primary one maybe wins or loses.
- There is an incentive to introduce thresholds.
- There are cases which discarding is inevitable.

Conclusions when margins are flexible



- There will be a pure equilibrium rather than a randomized one.
- Broker gains at expense of the secondary provider.
- Flexible margins with thresholds is the only stable equilibrium and the providers' gains in fixed margins scenario are mirage.

Research opportunities

- Considering cost of production and distribution
- Considering other types of contracts
- Considering different prices
- Stochastic sale modelling
- Providers' direct sale
- Providers' asymmetrical strategies

THANK YOU!