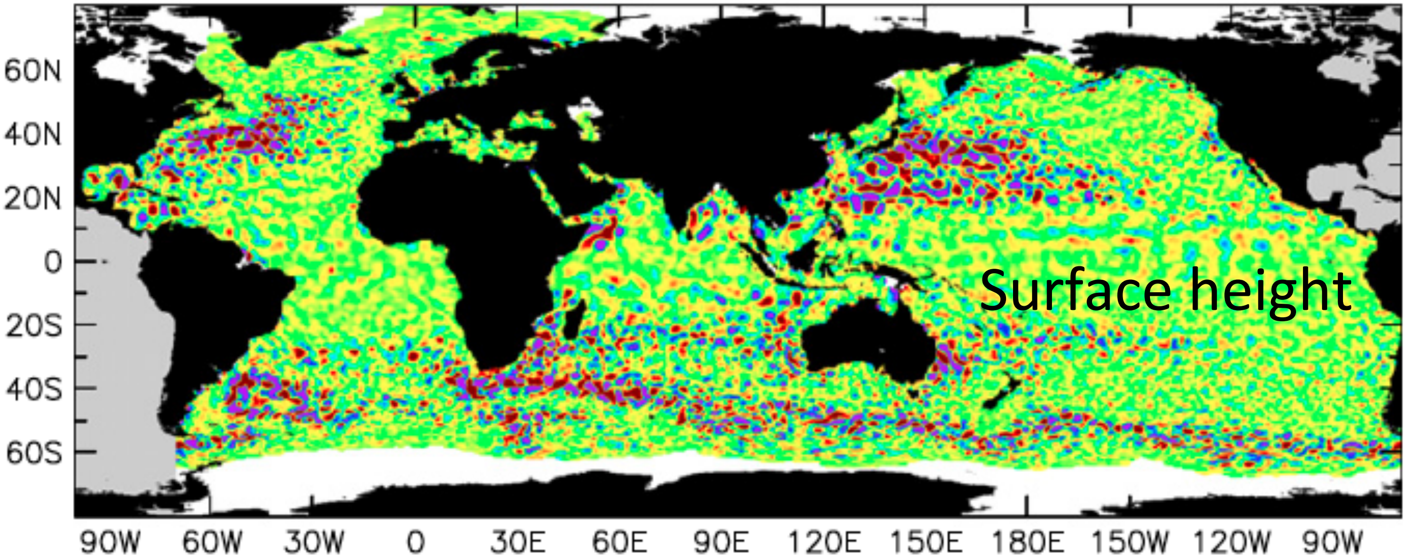
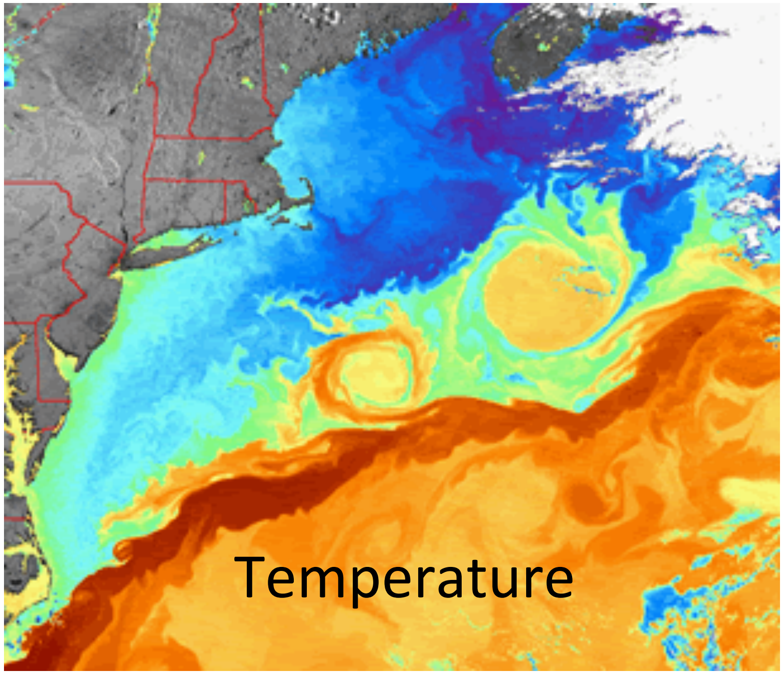
Stochastic Methods for Interpolating Satellite Imagery

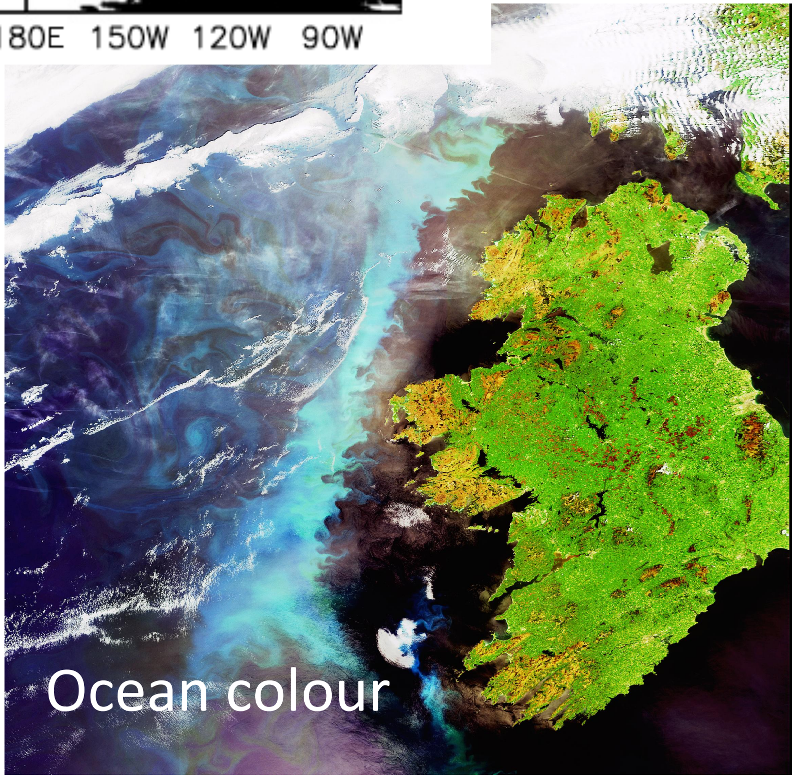
Shane R. Keating UNSW School of Mathematics & Statistics

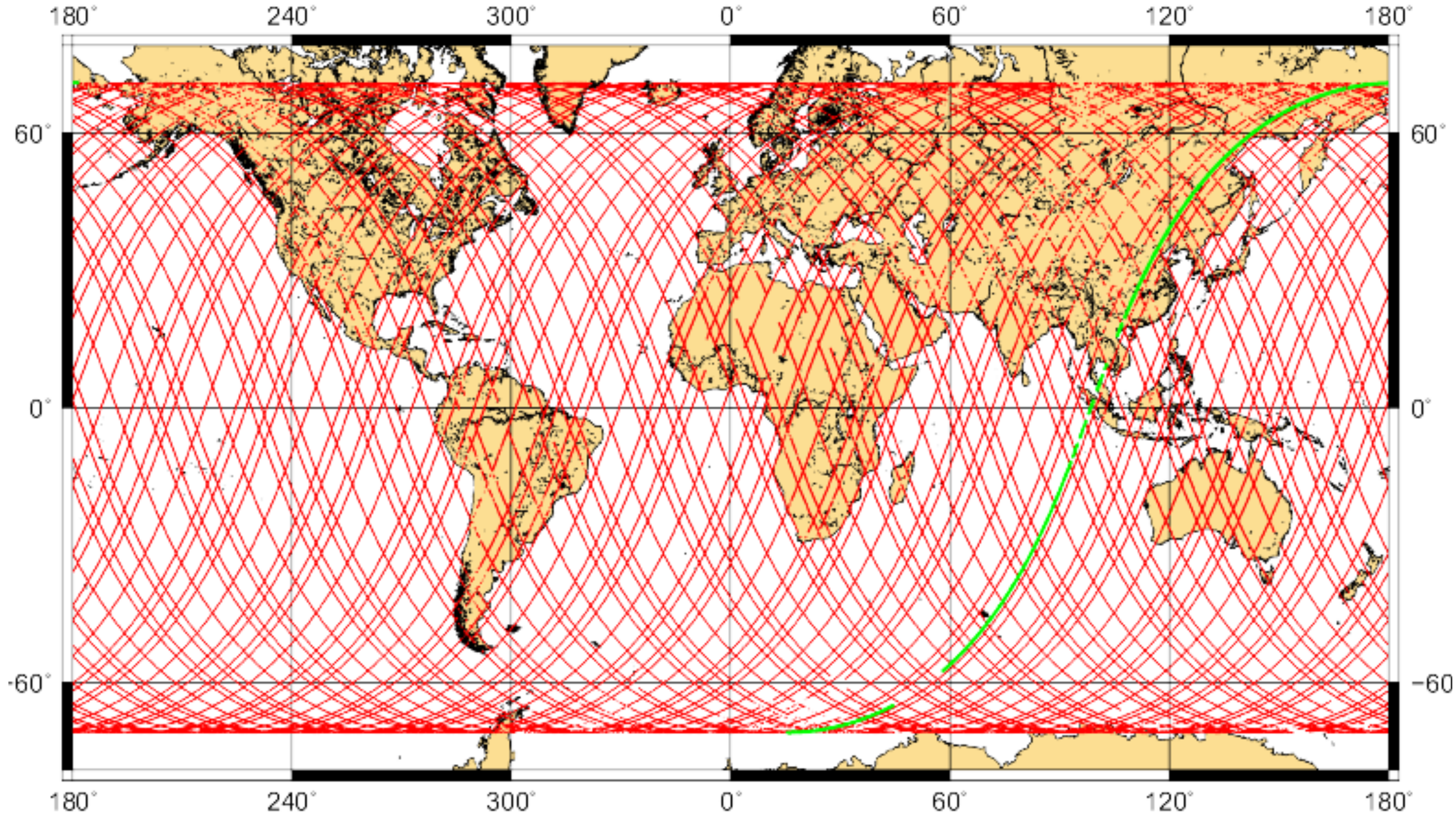
K. Shafer Smith, Andrew J. Majda NYU Courant Institute

NSW/ACT ANZIAM Meeting
Sydney 25 November 2015


60N
40N
20N
0
20S
40S
60S
90W
60W
30W
0
30E
60E
90E
120E
150E
180E
150W
120W
90W
Surface height


Temperature

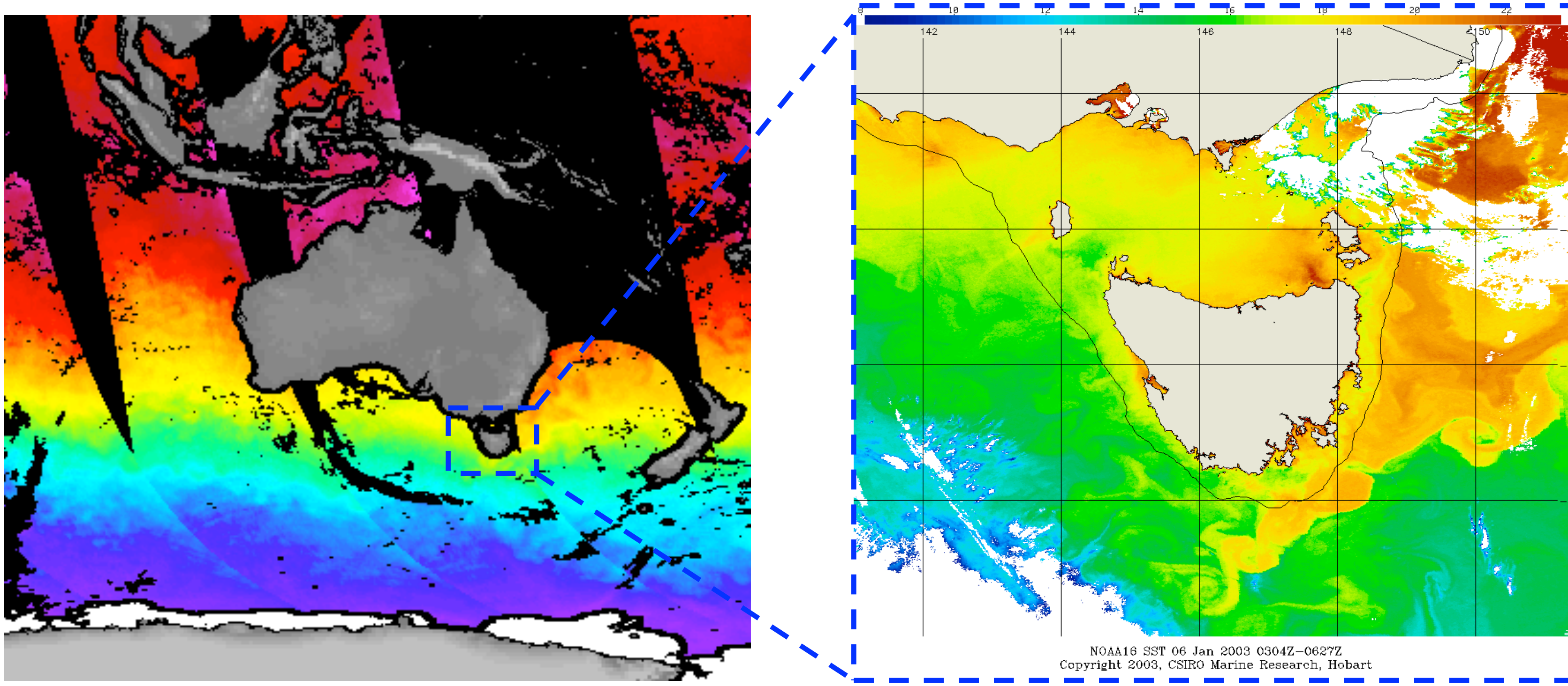

Ocean colour



Observing the ocean from space is challenging:

- *sparseness in horizontal*: many unresolved scales
- *sparseness in vertical*: observe upper ocean only
- *sparseness in time*: constrained by orbit
- *partial, noisy observations*: heterogeneous sampling, clouds

Microwave SST (AMSR-E) | Infrared SST (AVHRR)



Satellite imagery of sea-surface temperature observations:

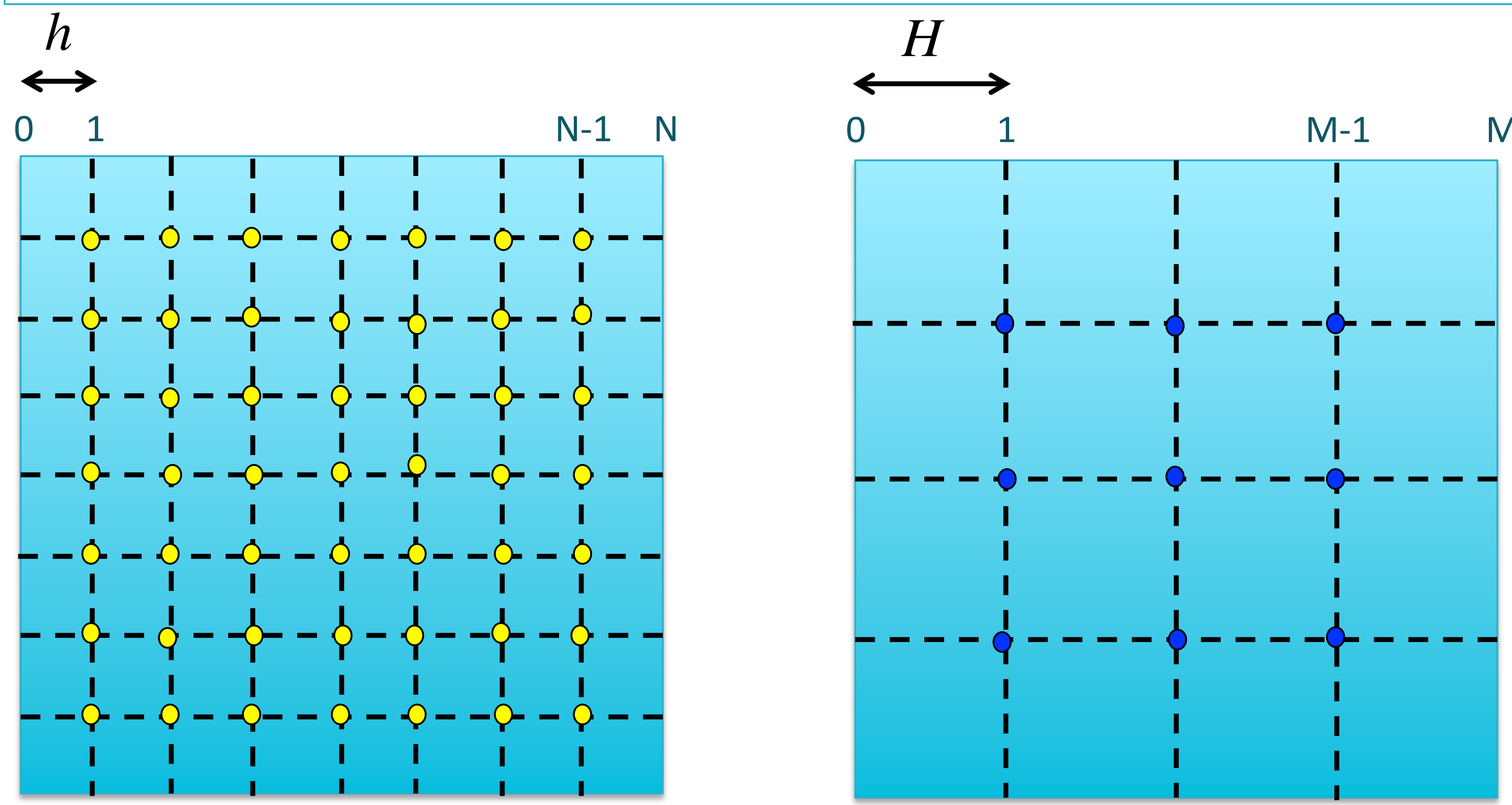
- **Microwave observations** have spatial resolutions of **20-50 km** and can **penetrate clouds**
- **Infrared observations** have spatial resolutions of **1-10 km** but are **obscured by clouds**

- Derive **superresolved images** by combining microwave observations with **statistical knowledge** from infrared images
- Exploit **spatial aliasing** of small scales by **coarse observations**

Original image

Subsampled image

Aliasing of sparse observations



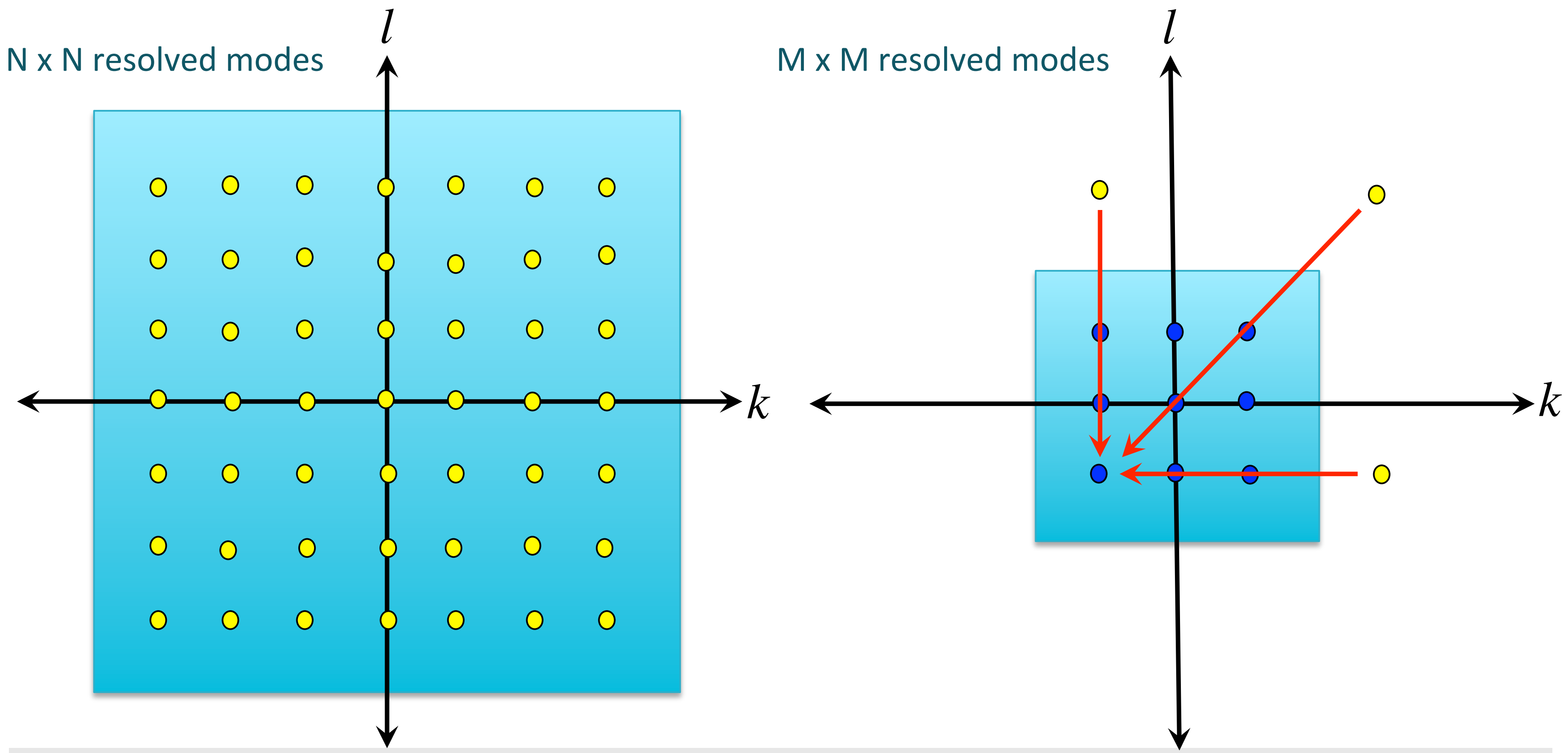
Fourier transform on **fine** grid:

$$\psi^{fine}_{\tilde{k},\tilde{l}} = \frac{1}{N^2}\sum_{m,n=1}^{N}\psi\,(mh,nh)e^{ih(m\tilde{k}+n\tilde{l})}$$

Fourier transform on **coarse** grid:

$$\psi^{coarse}_{k,l} = \frac{1}{M^2}\sum_{m,n=1}^{M}\psi\,(mH,nH)e^{iH(mk+nl)}$$

Aliasing of sparse observations



Coarse-grid modes are superposition of fine-grid modes in **same aliasing set.**

$$\psi_{k,l}^{coarse} = \sum_{\tilde{k},\tilde{l}} \psi_{\tilde{k},\tilde{l}}^{fine}$$

$$\tilde{k} \text{ mod M} = k$$
$$\tilde{l} \text{ mod M} = l$$

Aliasing of sparse observations

More generally, sample over footprint *G(x,y)*

$$\psi^{obs}(x,y) = \int G(x',y')\ \psi(x-x',y-y')\ dx'dy'$$

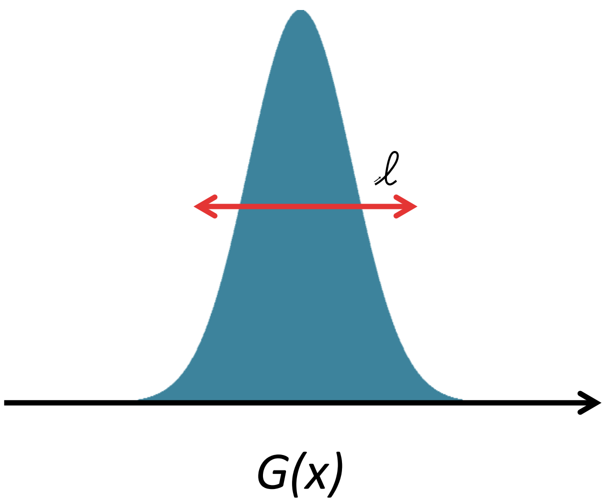
Coarse-grid Fourier transform is convolved with *spectral transfer function* $\hat{G}(k,l)$

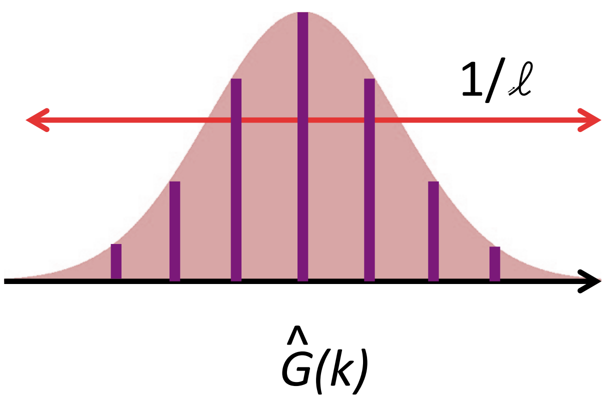
$$\psi^{obs}_{k,l} = \sum_{i,j} \hat{G}(k+iM, l+jM)\ \hat{\psi}_{k+iM,\, l+jM}$$

For a Gaussian sampling footprint of width ℓ, transfer function is a Gaussian of width $1/\ell$

$$G(x,y) = \frac{1}{2\pi\ell^2}\exp\left[-\left(x^2+y^2\right)/2\ell^2\right]$$

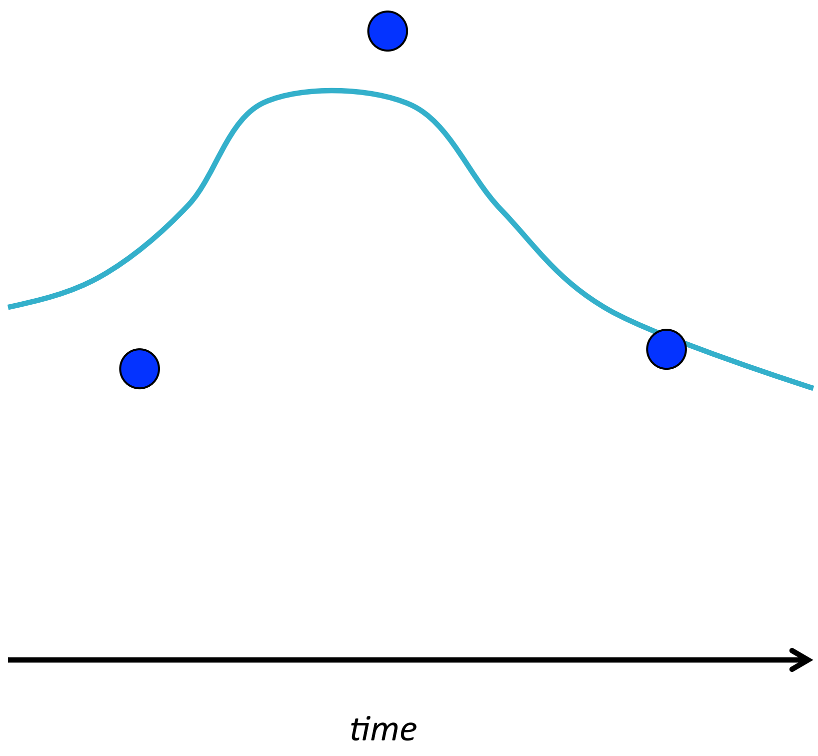
$$\hat{G}(p,q) = \exp\left[-2\pi^2\left(p^2+q^2\right)\ell^2/2L^2\right]$$





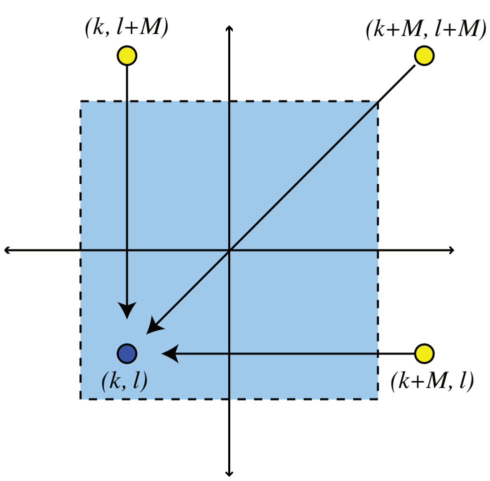
Filtering sparse observations

Data assimilation or filtering seeks the *best-guess estimate* of the *state of the system* by combining *noisy, incomplete observations* with an *internal forecast model.*



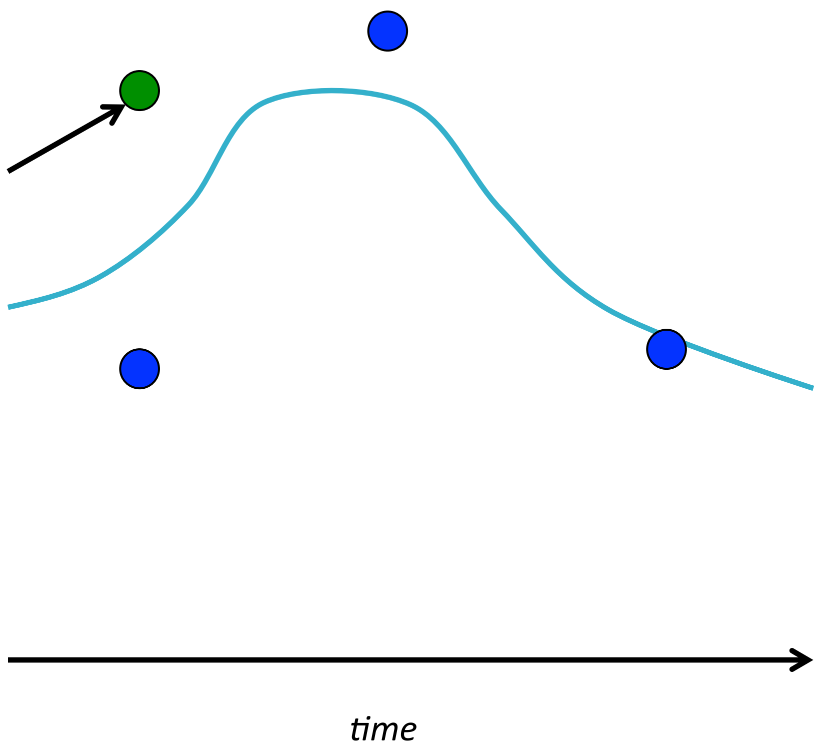
M x M observations of each resolved mode + aliased modes

$$\psi_{k,l}^{obs} = \sum_{\tilde{k},\tilde{l}} \psi_{\tilde{k},\tilde{l}}^{true} + \sigma_{k,l}^{obs}$$



Filtering sparse observations

Data assimilation or filtering seeks the *best-guess estimate* of the *state of the system* by combining *noisy, incomplete observations* with an *internal forecast model.*



1. Forecast step:

Make prediction for *N x N* modes using quasi-linear stochastic model.

$$\partial_t \hat{\theta} = -(\gamma - \mathrm{i}\omega)\hat{\theta}(t) + \sigma \dot{W}(t)$$

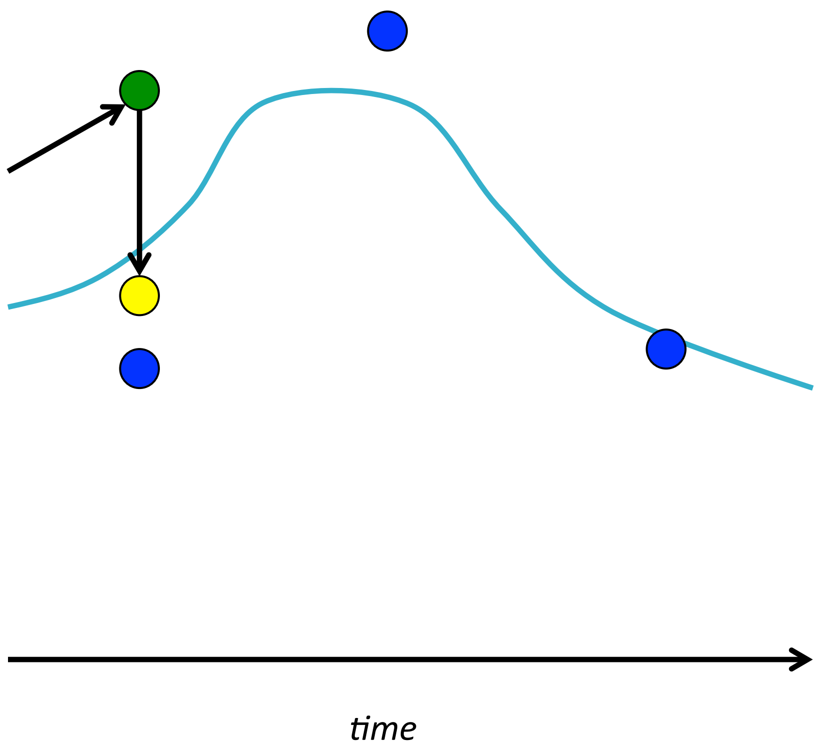
Forecast mean and covariance:

$$\langle \theta \rangle, \quad R_{pq} = \langle \theta_p^* \theta_q \rangle$$

Tune parameters to give correct energy and timescales estimated from **infrared observations**.

Filtering sparse observations

Data assimilation or filtering seeks the *best-guess estimate* of the *state of the system* by combining *noisy, incomplete observations* with an *internal forecast model.*



2. Update step:

Combine *N x N* prediction (-) with *M x M* observation (~) using **Kalman filter** solution:

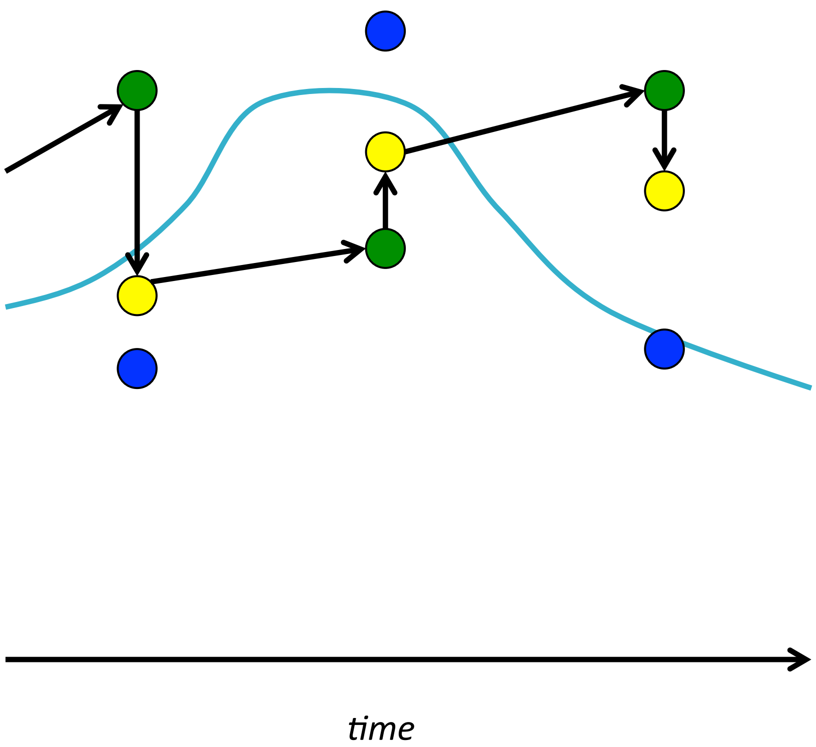
$$\langle \theta_+ \rangle = (1 - KG)\langle \theta_- \rangle + K\tilde{\theta}$$

$$R_+ = (1 - KG)R_-$$

Optimal solution when dynamics and observation operator are linear with unbiased uncorrelated Gaussian noise.

Filtering sparse observations

Data assimilation or filtering seeks the *best-guess estimate* of the *state of the system* by combining *noisy, incomplete observations* with an *internal forecast model.*



2. Update step:

Combine *N x N* prediction (-) with *M x M* observation (~) using **Kalman filter** solution:

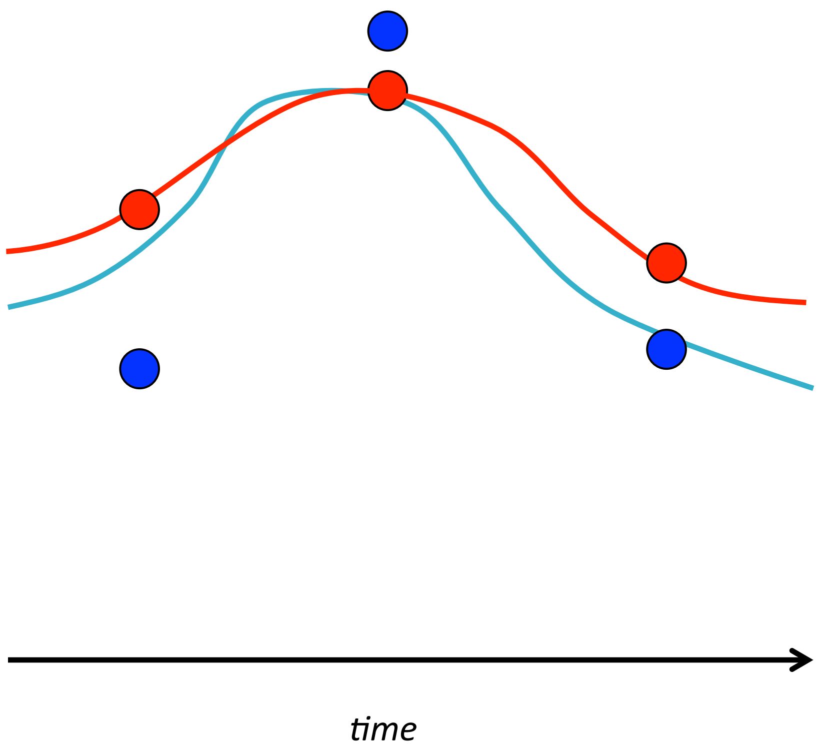
$$\langle \theta_+ \rangle = (1 - KG)\langle \theta_- \rangle + K\tilde{\theta}$$

$$R_+ = (1 - KG)R_-$$

Optimal solution when dynamics and observation operator are linear with unbiased uncorrelated Gaussian noise.

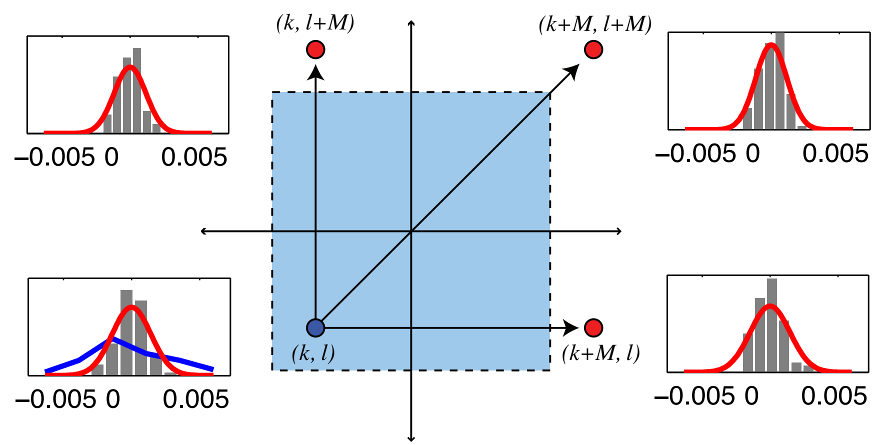
Filtering sparse observations

Data assimilation or filtering seeks the *best-guess estimate* of the *state of the system* by combining *noisy, incomplete observations* with an *internal forecast model.*

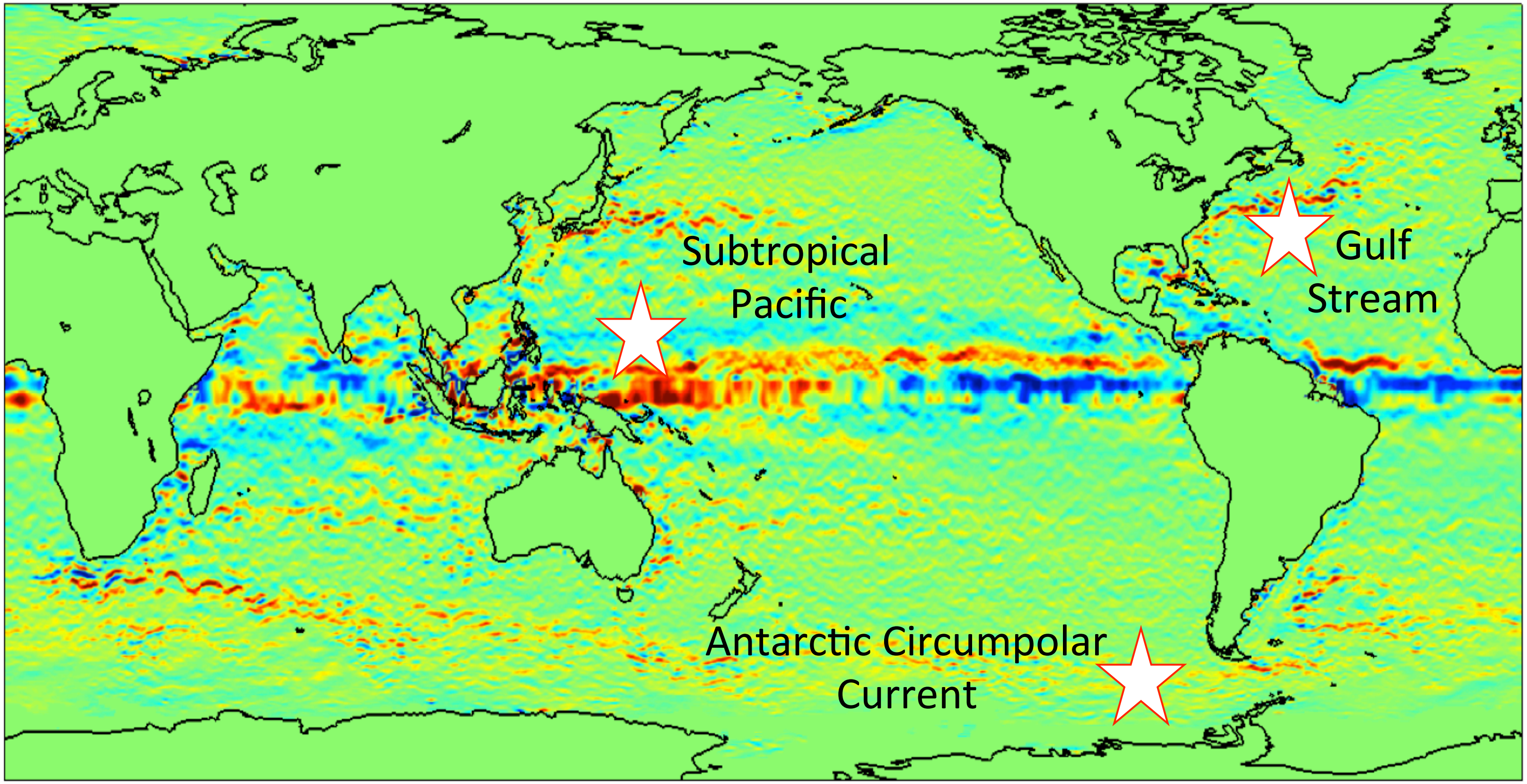


3. Smoothing step:

Apply **Rauch-Tung-Straub smoother** to remove unphysical jumps.

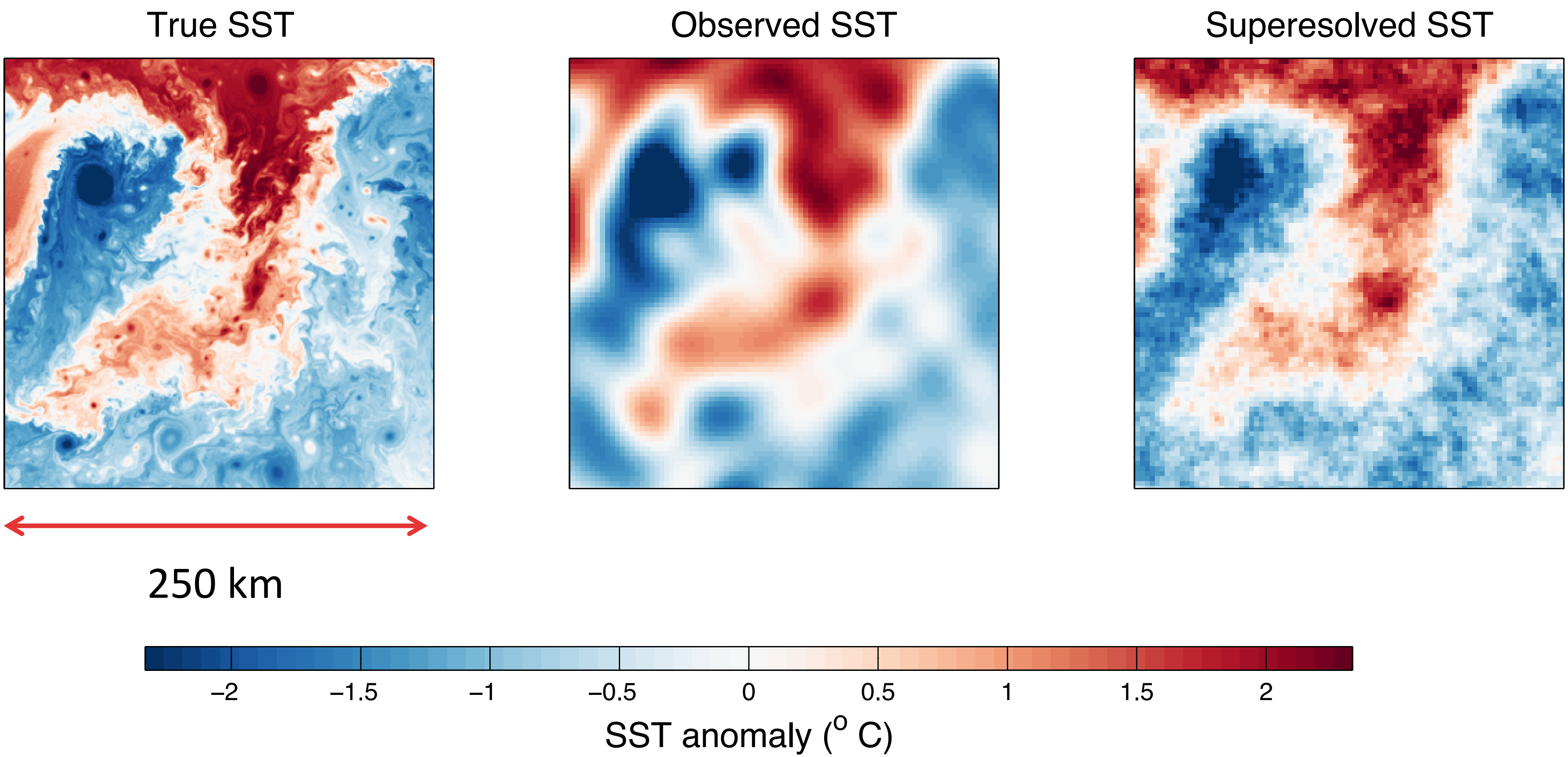


Resulting **superresolved estimate** is a pdf with an **effective resolution** given by model, not observations.



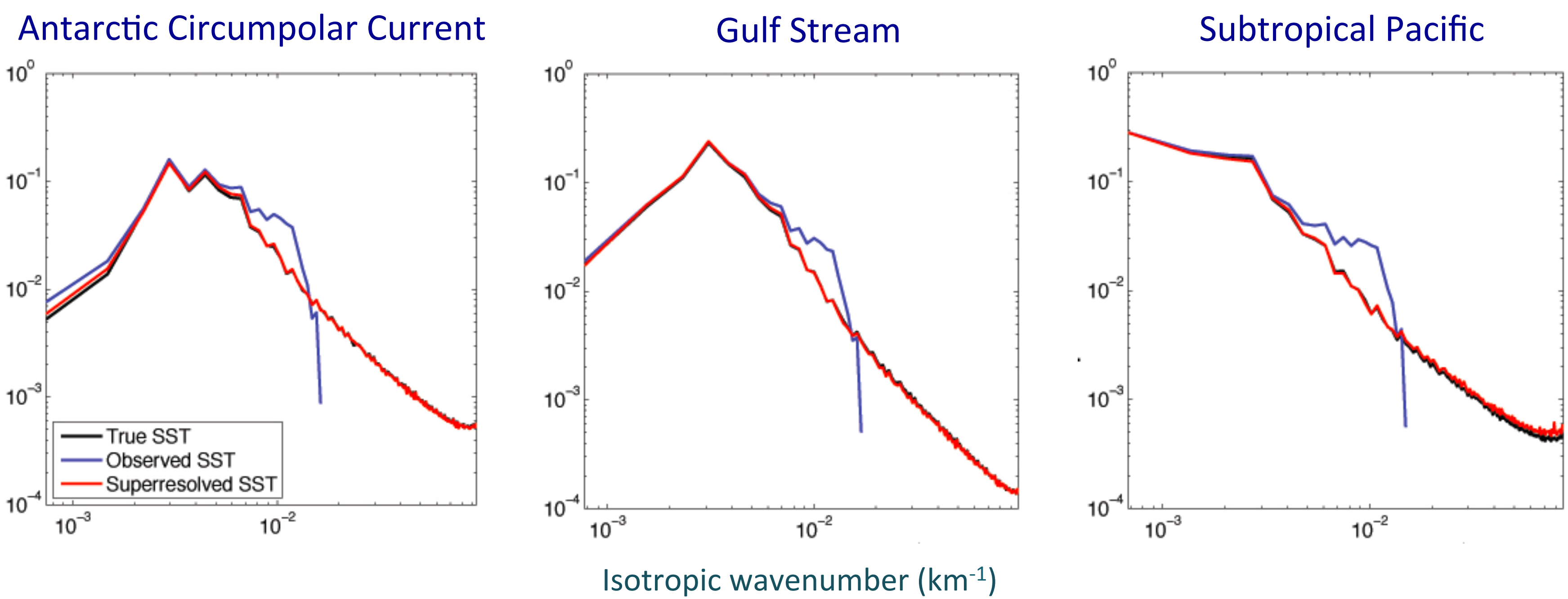
- Test in **ocean simulations** driven by Forget (2010) hydrography.
- Assume that **density anomalies** are dominated by temperature.
- Synthetic daily temperature observations over a 90-day period with both **microwave (40 km) and infrared (5 km) resolutions**.
- Infrared observations used to learn **stochastic parameters**.

Sea-surface temperature (SST) snapshots: Subtropical Pacific



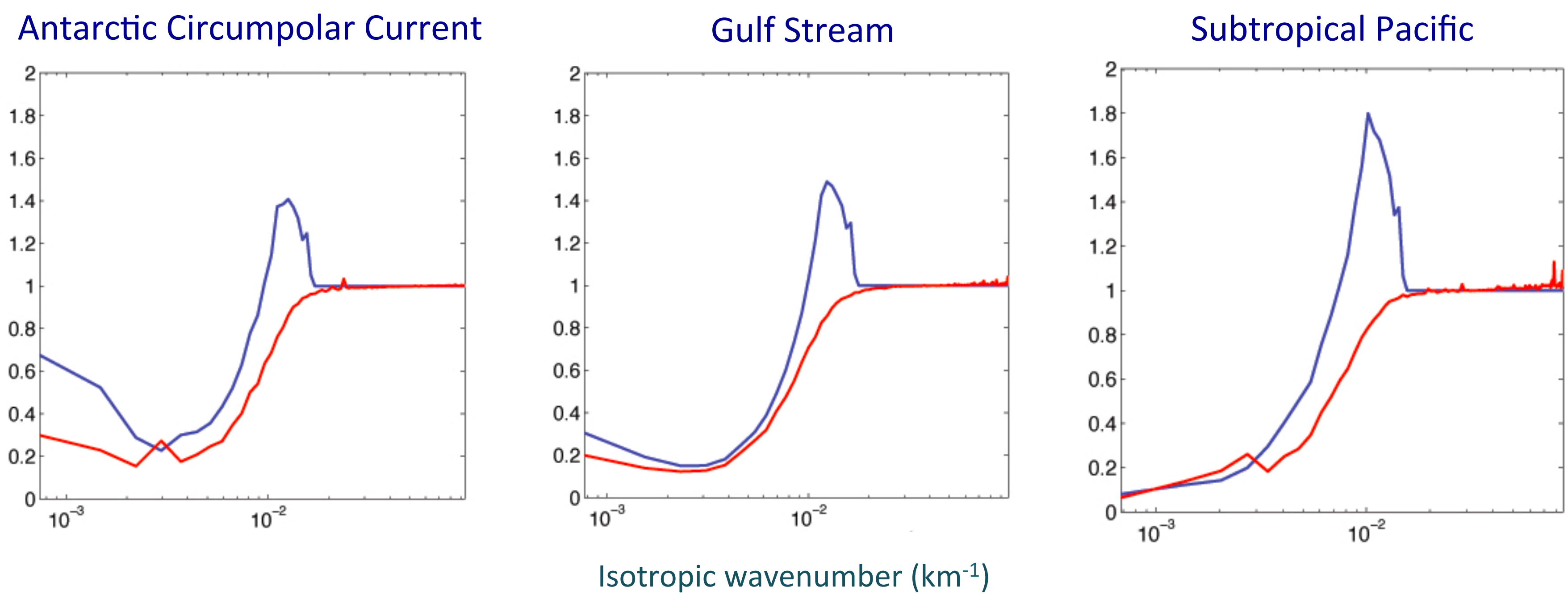
$$\theta_{kl} = \langle \theta_{kl} \rangle + A(k,l)X, \qquad A^*(k,l)A(k,l) = R(k,l)$$

Temperature variance spectrum: $\left\langle |\theta(k)|^2 \right\rangle$



- **Effect of aliasing** can be seen in spurious variance in observations near the limit of resolution
- Super-resolved estimate correctly **redistributes variance** to small scales

RMS error: $\left\langle \left|\theta(k)-\theta^{true}(k)\right|^2 \right\rangle^{1/2} \Big/ \left\langle \left|\theta^{true}(k)\right|^2 \right\rangle^{1/2}$



- **Effect of aliasing** can be seen in spurious variance in observations near the limit of resolution
- Super-resolved estimate correctly **redistributes variance** to small scales

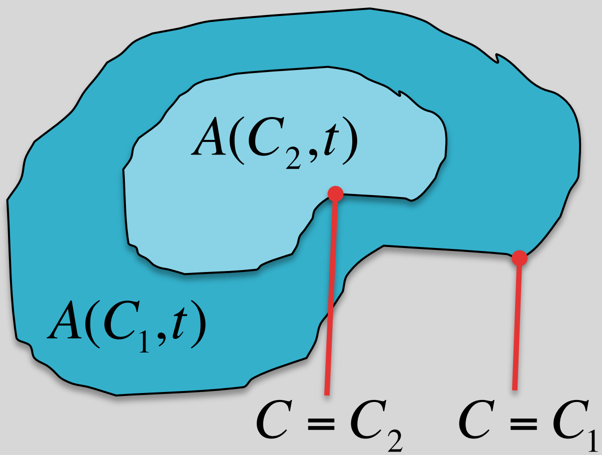
Eddy diffusion across tracer contours

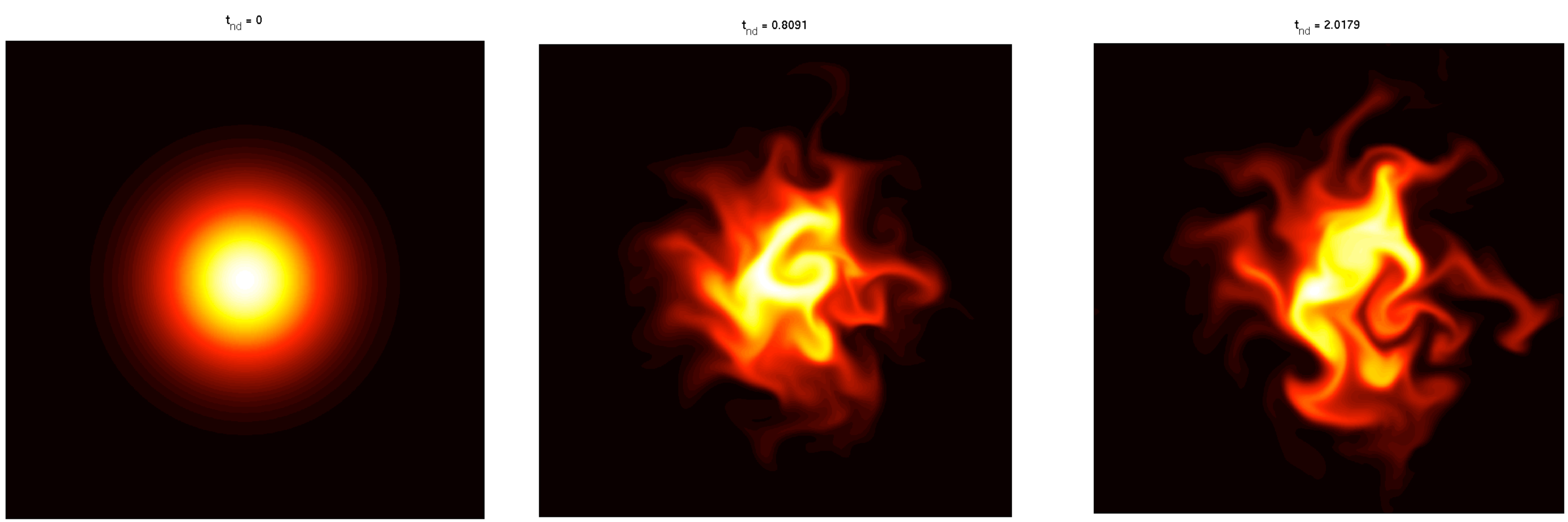
- Tracer coordinates : area enclosed by *C*

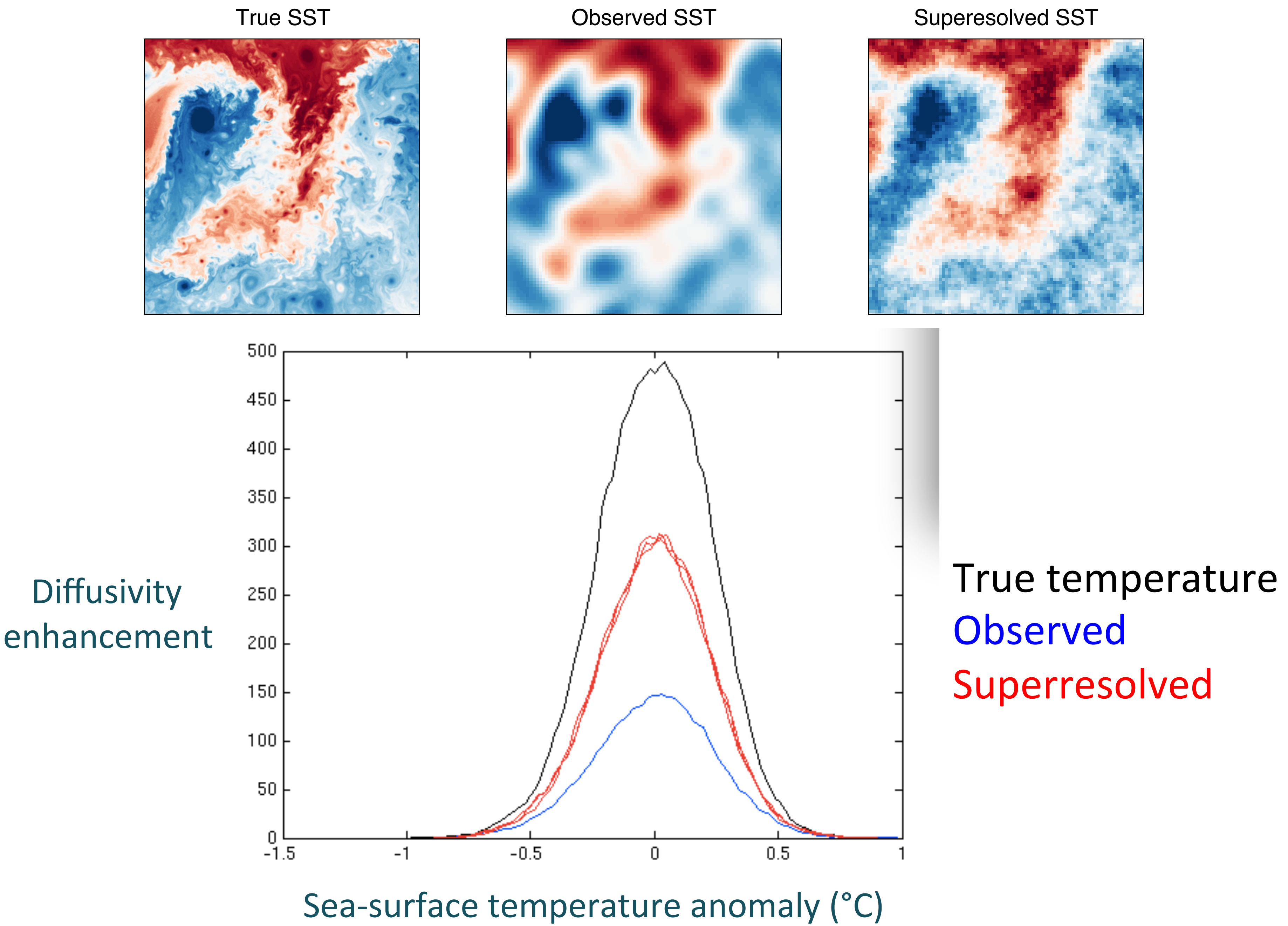
$$\partial_t C = \partial_A \left(K_{eff} \partial_A C \right)$$

- ***Effective diffusivity*** *(Nakamura 1996):*

$$K_{eff}(C,t) = K \oint \left| \nabla C^* \right| dl \oint \left| \nabla C^* \right|^{-1} dl$$







True SST
Observed SST
Superesolved SST
Diffusivity enhancement
500
450
400
350
300
250
200
150
100
50
0
-1.5
-1
-0.5
0
0.5
1
Sea-surface temperature anomaly (°C)
True temperature
Observed
Superresolved

Conclusions

AGU PUBLICATIONS

JGR

Journal of Geophysical Research: Oceans

RESEARCH ARTICLE

10.1002/2014JC010357

Upper ocean flow statistics estimated from superresolved sea-surface temperature images

Shane R. Keating[1] and K. Shafer Smith[2]

[1]School of Mathematics and Statistics, University of New South Wales, Sydney, New South Wales, Australia, [2]Center for Atmosphere-Ocean Science, Courant Institute of Mathematical Sciences, New York University, New York, New York, USA

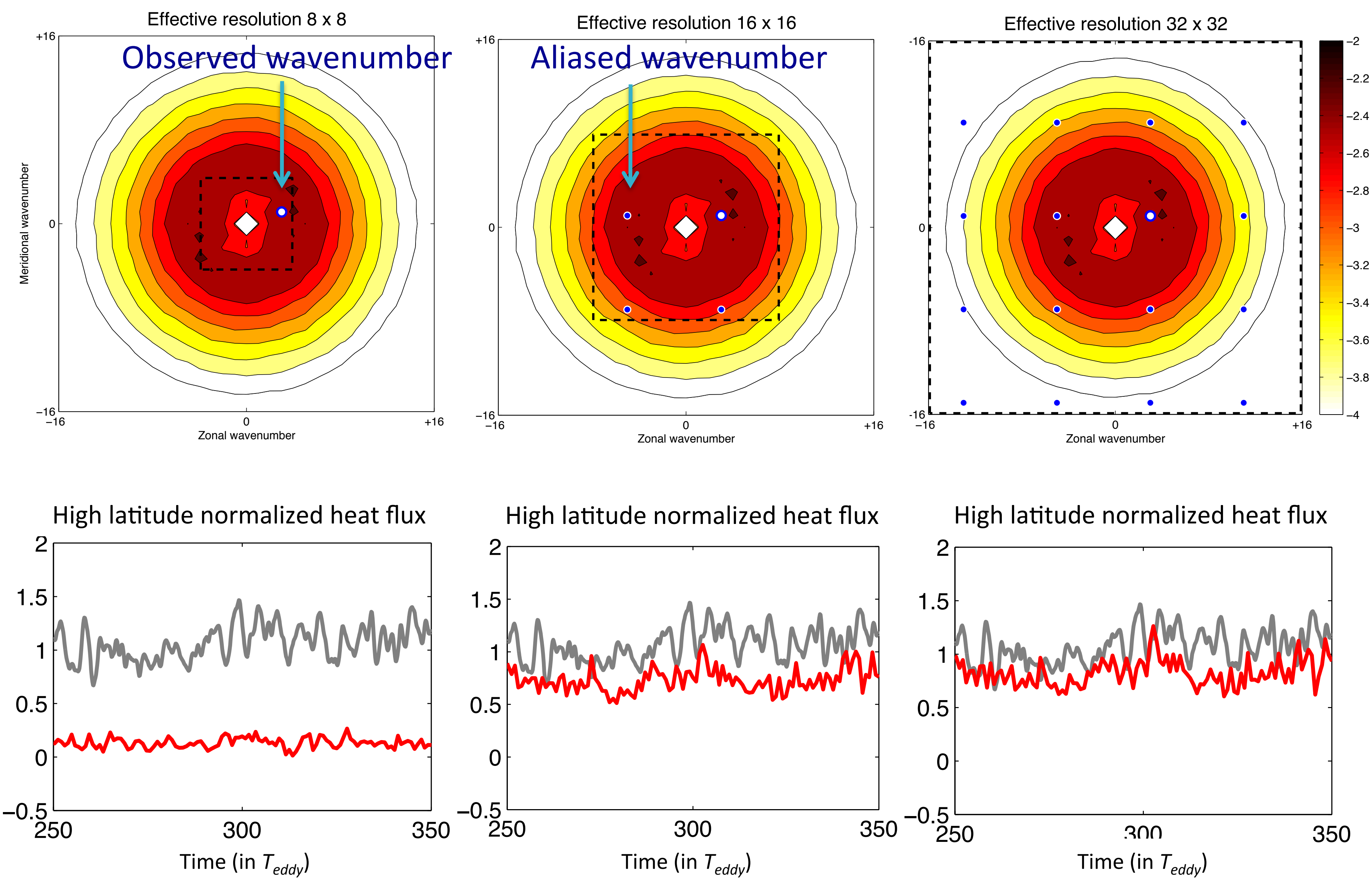
Key Points:

- The resolution of microwave SST images is increased using a statistical model
- The model is based upon statistics learned from intermittent infrared

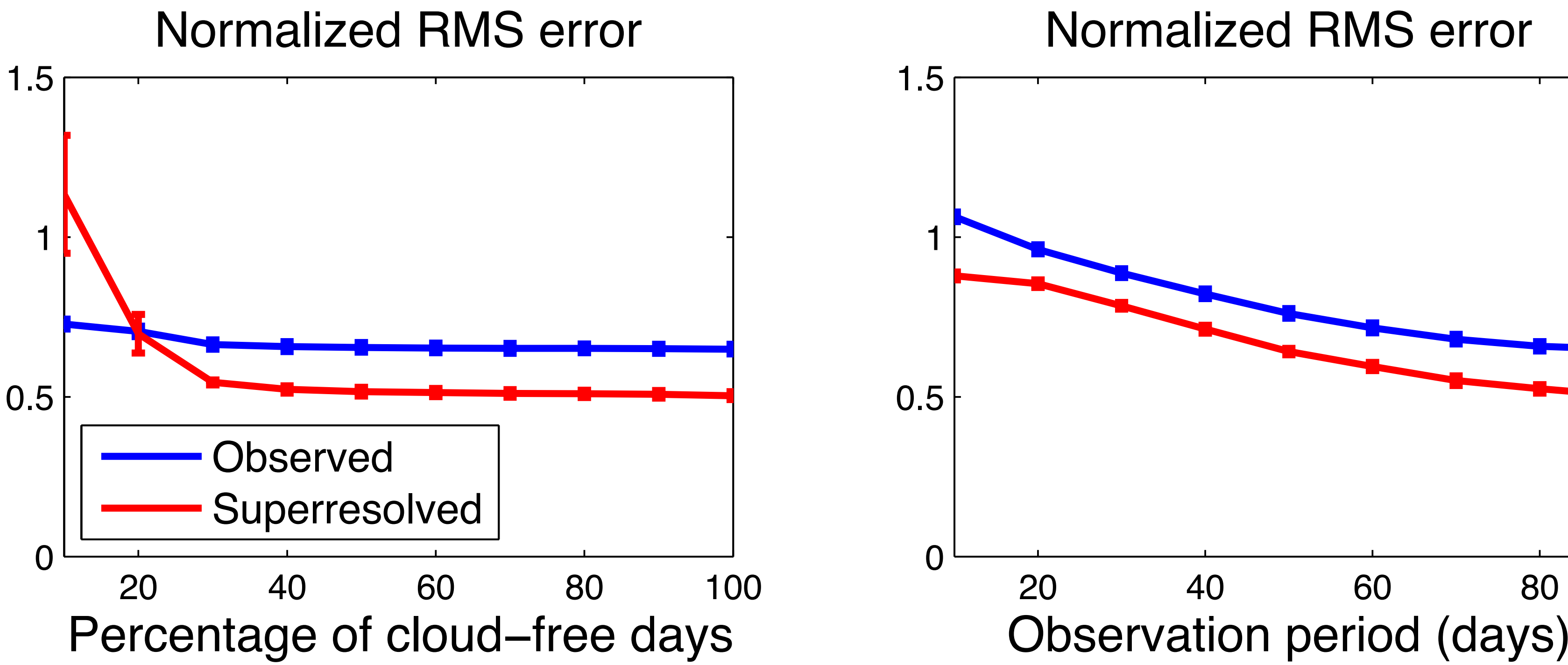
---

- Combine coarse-resolution microwave images with a simple statistical model to construct **super-resolved images.**
- Stochastic model based upon statistical information from intermittent infrared observations.
- Keating, S.R. and Smith, K.S. (2015) *J. Geophys. Res.* 120: 1-18

Estimating poleward ocean heat flux



Sensitivity to **clouds and observing period**:



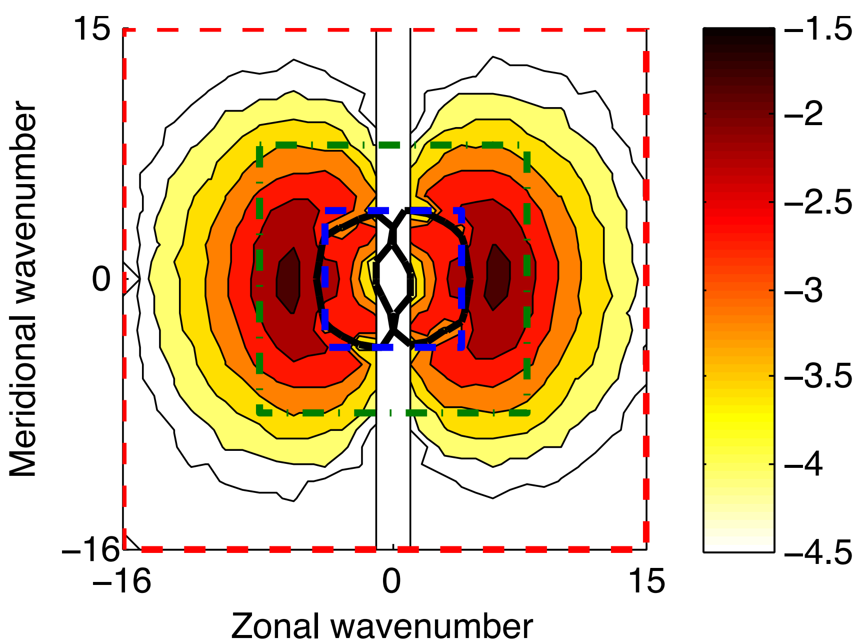
- **Accuracy of small-scale statistics** calculated using high-resolution images depends on quality of data
- **Model effect of imperfect data** by randomly discarding frames ("clouds") or shortening observing period

Eddy heat flux in the Phillips model

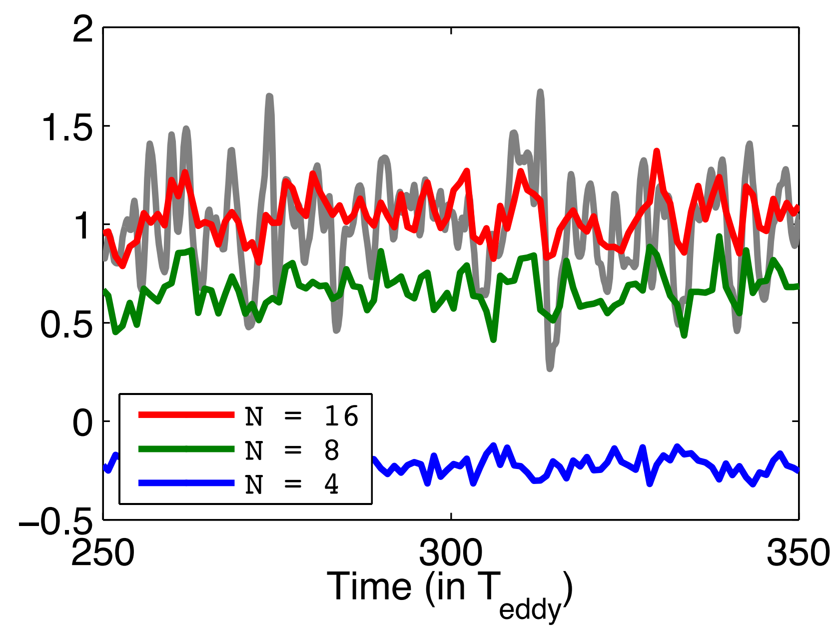
$$\text{Heat flux} = \langle v_1 \tau \rangle = -\sqrt{d_1 d_2} \langle \psi_2 \partial_x \psi_1 \rangle$$

- Explicitly a function of *both* upper and lower layers
- Sensitive to horizontal spatial resolution

Heat flux spectrum



Optimally interpolated heat flux



Stochastic Forecast Model

$$\dot{u}_\alpha(t) = m(t)u_\alpha(t) + a(t) + \hat{\sigma}_\alpha \dot{W}_\alpha(t) \qquad \alpha = \{k, l, \mu\}$$

Offline parameter estimation

$$m(t) = m_0 = -\gamma_\alpha + \mathrm{i}\omega_\alpha$$

$$a(t) = a_0 = 0$$

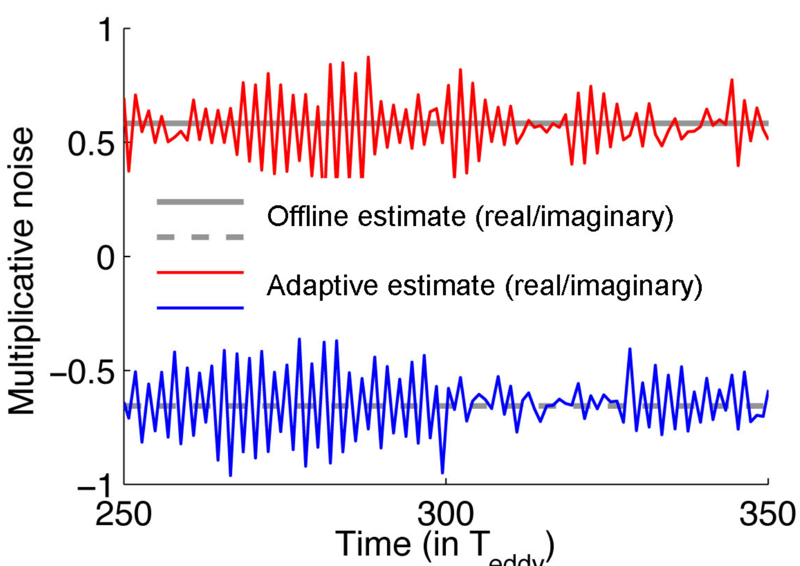
Regression fit to ***time-mean*** energy and correlation time.

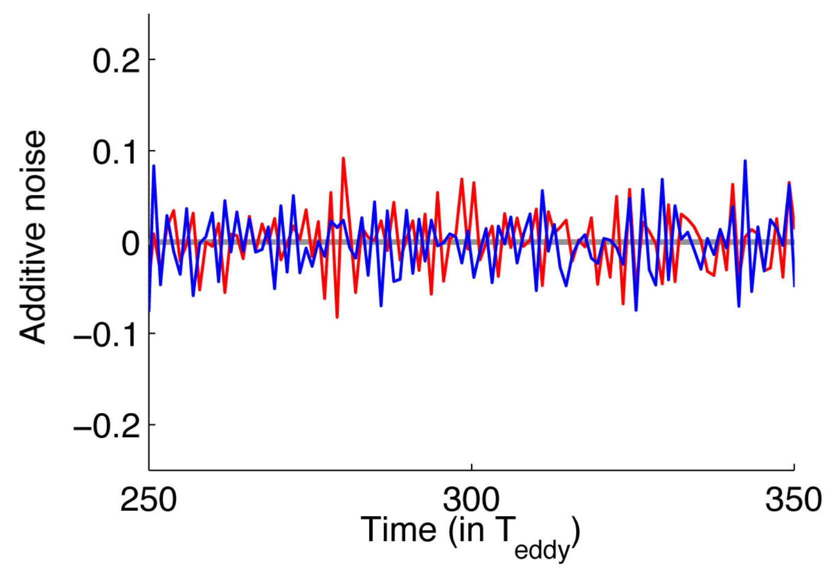
Adaptive parameter estimation

$$\dot{m}(t) = -\lambda_m\big(m(t) - m_0\big) + \sigma_m \dot{W}_m(t)$$

$$\dot{a}(t) = -\lambda_a\big(a(t) - a_0\big) + \sigma_a \dot{W}_a(t)$$

High filtering skill for ***broad range*** of parameters.





Stochastic Superresolution