Wave interaction with a floating circular flexible porous membrane in a two-layer fluid

Harekrushna Behera[1], Siluvai Antony Selvan[1] and Vinay Kumar Gupta[1,2]

[1]SRM Research Institute & Department of Mathematics, SRM Institute of Science and Technology, Kattankulathur 603203, Tamil Nadu, India

[2]Mathematics Institute, University of Warwick, Coventry CV4 7AL, UK.

E-mail: hkb.math@gmail.com; antony.selvan22@gmail.com; vinay.gupta@warwick.ac.uk

Highlights

- The surface gravity wave interaction with a floating flexible circular porous membrane in a two-layer fluid is studied using the linearized theory of water waves.
- The boundary value problem is solved using the matched eigenfunction expansion method, and Darcy's law is used for wave past porous structure.
- The vertical wave force acting on the membrane, surface and interface elevations, deflection of the membrane and flow distribution around the structure in a two-layer fluid are computed and analyzed for various structural and wave parameters.
- The study reveals that the density ratio, porous-effect parameter, radius of the membrane and depth ratio are the major parameters for reducing wave force on the structure and creating calm region in the lee side of the structure. Thus, a floating circular flexible porous membrane can be used as a cost-effective coastal structure to protect marine facilities.

1 Introduction

In the recent years, there has been a significant progress in the literature on wave interaction with floating/submerged horizontal/vertical impermeable/permeable structures as breakwaters. Furthermore, the use of submerged horizontal porous structures has become increasingly popular as compared to vertical structures as the former does not obstruct the ocean view, which is essential for development of recreational and residential shore. Behera and Sahoo [1] analyzed the problem of wave scattering and trapping by a submerged horizontal porous plate using the matched eigenfunction expansion method. They observed that wave reflection and transmission are reduced significantly due to the dissipation of wave energy through the submerged plate. Using the coupled boundary element and finite element methods, Meylan et al. [2] investigated the wave past a porous elastic structure of arbitrary shape. Koley et al. [3] developed a system of Fredholm type integral equations using Green's function technique for analyzing wave scattering by a floating flexible plate in both finite and infinite water depth. In general, the ocean has several layers of fluid due to the mixing of chemical and solar radiation. Such stratifications account for preciseness in studying wave scattering problems. Mondal et al. [4] investigated the deflection of a floating flexible plate/membrane in a two-layer fluid. To the best of authors knowledge, there is no study available in the literature on wave interaction with floating circular flexible porous membrane/plate in a two-layer fluid, although the use of these structures as breakwater in coastal regions is ubiquitous in protecting various marine facilities. This has formed the motivation of the present study, which aims to look into wave scattering by a floating circular flexible porous membrane in a two-layer fluid and this study can be easily extended to wave scattering/trapping by a floating circular flexible porous plate in the presence/absence of an undulated bottom in a single-/two-layer fluid.

2 Mathematical formulation

The problem is studied in cylindrical polar coordinates (r, θ, z) with horizontal plane being the $r\theta$-plane and vertically downward direction the positive z-axis. Fig. 1 displays a schematic diagram depicting a thin floating circular flexible porous membrane of radius a and thickness d floating on a two-layer fluid at $z = 0$. The center of the membrane is taken at origin. The two immiscible fluids of densities ρ_1 and ρ_2 (where $\rho_1 < \rho_2$) are confined in the regions $0 \leq z \leq h$ (upper layer) and $h \leq z \leq H$

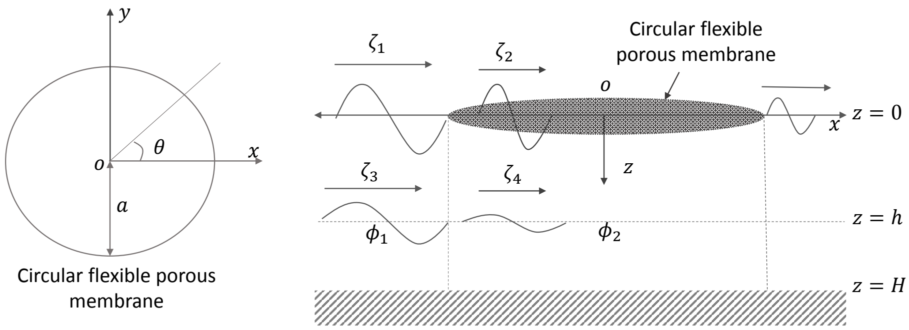


Figure 1: Schematic diagram of wave scattering by a floating circular flexible porous membrane in a two-layer fluid.

(lower layer), respectively. In both the layers, the fluid flow is assumed to be incompressible, inviscid, irrotational and to follow simple harmonic motion. Let the time dependent velocity potential is given by $\Phi(r,\theta,z,t) = \mathrm{Re}[\phi(r,\theta,z)\,\mathrm{e}^{\mathrm{i}\omega t}]$ with ω being the angular velocity. Furthermore, the spatial velocity potential $\phi_j(r,\theta,z)$ satisfies Laplace equation in the j^{th} region which is given by

$$\nabla^2_{r\theta}\phi_j + \partial^2_z\phi_j = 0 \quad \text{with} \quad \nabla^2_{r\theta} \equiv \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\right) \quad \text{and} \quad \partial^2_z \equiv \frac{\partial^2}{\partial z^2} \text{ for } j = 1,2,3,4. \tag{1}$$

The elevation in both the surface and internal modes in the open water and membrane covered region is given by

$$\zeta_j(r,\theta,z,t) = \eta_j(r,\theta,z)\,\mathrm{e}^{-\mathrm{i}\omega t} \quad \text{for} \quad j = 1,2,3,4, \tag{2}$$

where η_1 and η_2 are the amplitudes of the free surface elevation and deflection of the membrane in surface mode, respectively. On the other hand, η_3 and η_4 denote the amplitudes of the elevations in internal mode for open water and membrane covered regions, respectively. The boundary conditions at rigid sea bottom, free surface and membrane covered regions are given by

$$\partial_z\phi_j = 0 \quad \text{at} \quad z = H \quad \text{for} \quad j = 1,2, \tag{3}$$

$$\left.\begin{array}{l} \partial_z\phi_1 + L\phi_1 = 0, \\ \left(T\nabla^2_{r\theta} - \rho g + \rho_m d_1\omega^2\right)\left(\partial\phi_{2z} - \mathrm{i}k_0 G\phi_2\right) = \rho\omega^2\phi_2 \end{array}\right\} \text{ at } z = 0, \tag{4}$$

where T, ρ_m, G and k_0 are the tensile force, membrane density, porous-effect parameter and progressive wavenumber, respectively, and $L = \omega^2/g$ with g being the gravitational acceleration. At interface $z = h$, the conditions are given by

$$\partial_z\phi_j\Big|_{z=h^-} = \partial_z\phi_j\Big|_{z=h^+}, \quad s\left(\partial_z\phi_j + L\phi_j\right)\big|_{z=h^-} = \left(\partial_z\phi_j + L\phi_j\right)\big|_{z=h^+}, \tag{5}$$

where $s = \rho_1/\rho_2$ is the density ratio. The far-field condition is given by the Sommerfeld radiation condition as

$$\lim_{r\to\infty}\sqrt{r}\left\{\frac{\partial(\phi - \phi_{\mathrm{inc}})}{\partial r} - \mathrm{i}k_0(\phi - \phi_{\mathrm{inc}})\right\} = 0, \tag{6}$$

where

$$\phi_{inc} = -\left(\frac{\mathrm{i}g\alpha_0}{2\omega}\right)\frac{\cosh[k_0(h-z)]}{\cosh[k_0 h]}\sum_{m=0}^{\infty}\epsilon_m J_m(k_0 r)\cos(m\theta) \quad \text{for} \quad 0 < \theta < 2\pi.$$

Here, α_0 is the known incident wave amplitude, $\epsilon_m = 1$ for $m = 0$, $\epsilon_m = 2\mathrm{i}^m$ for $m = 1,2,3,\ldots$, and J_m is the Bessel function of first kind of order m. The velocity and pressure matching conditions along the interface of porous membrane and open water is given by

$$\phi_1 = \phi_2 \quad \text{and} \quad \partial_r\phi_1 = \partial_r\phi_2, \quad \text{on} \quad r = a \quad \text{for} \quad 0 < z < h,\ 0 < \theta \le 2\pi, \tag{7}$$

The free edge conditions of porous membrane is given as follows

$$\partial_r\left(\partial_z\phi_2 - \mathrm{i}k_0 G\phi_2\right) = 0, \quad \text{on} \quad r = a \quad \text{for} \quad z = 0,\ 0 < \theta \le 2\pi, \tag{8}$$

3 Method of solution

The spatial velocity potentials in the open water $\phi_1(r,\theta,z)$ and membrane covered regions $\phi_2(r,\theta,z)$ satisfying Eqs. (1),(3)–(4) are given by

$$\phi_1(r,\theta,z) = \sum_{m=0}^{\infty}\left[\sum_{n=I}^{II}\left(\frac{-\mathrm{i}g\alpha_0}{2\omega}\right)\epsilon_m J_m(k_n r) + \sum_{n=I,II,1}^{\infty} A_{mn}H_m(k_n r)\right] f_{1n}(z)\cos(m\theta), \tag{9}$$

$$\phi_2(r,\theta,z) = \sum_{m=0}^{\infty}\left[\sum_{n=I,II,III,1}^{\infty} B_{mn}J_m(p_n r)\right] f_{2n}(z)\cos(m\theta), \tag{10}$$

where H_m is the Hankel function of first kind of order m, and A_{mn} and B_{mn} are the unknown constants. The open water and membrane covered vertical eigenfunctions are given by

$$f_{1n}(z) = \begin{cases} -\dfrac{\sinh\{k_n(H-h)\}[k_n\cosh(k_n z) - L\sinh(k_n z)]}{k_n\sinh(k_n h) - L\cosh(k_n h)} & \text{for} \quad 0 \le z \le h, \\ \cosh\{k_n(H-z)\} & \text{for} \quad h \le z \le H. \end{cases} \tag{11}$$

$$f_{2n}(z) = \begin{cases} V(p_n)\left[p_n S\cosh(p_n z) - (L - \mathrm{i}k_0 GS)\sinh(p_n z)\right] & \text{for} \quad 0 \le z \le h, \\ \cosh\{p_n(H-z)\} & \text{for} \quad h \le z \le H, \end{cases} \tag{12}$$

$$\text{where} \quad V(p_n) = -\frac{\sinh\{p_n(H-h)\}}{p_n S\sinh(p_n h) - (L - \mathrm{i}k_0 GS)\cosh(p_n h)} \quad \text{and} \quad S = Qp_n^2 - M + 1, \tag{13}$$

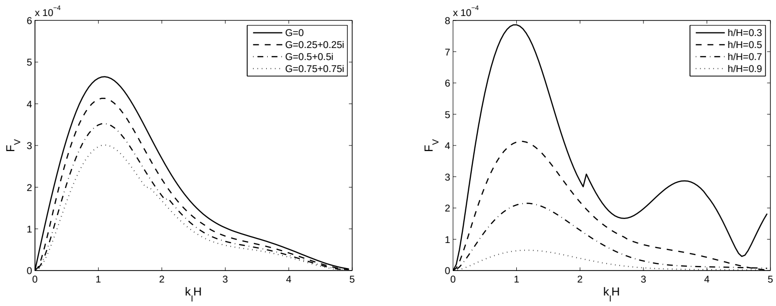
with $Q = T/\rho g,\ M = m_m\omega^2/\rho g$ and $m_m = \rho_m d$. The eigenvalue p satisfies dispersion relation in the membrane covered region

$$\begin{aligned} &L^2\left[s\tanh(ph)\tanh\{p(H-h)\} + 1\right] - pL\Big[M\Big\{s\tanh\{p(H-h)\} + \tanh(ph)\Big\} \\ &\quad + (1-s)\tanh\{p(H-h)\}\Big] + p^2M(1-s)\tanh\{p(H-h)\}\tanh(ph) + \mathrm{i}k_0G \\ &\qquad \Big[pM(1-s)\tanh\{p(H-h)\} - ML\Big\{1 + s\tanh\{p(H-h)\}\tanh(ph)\Big\}\Big] = 0. \end{aligned} \tag{14}$$

It may be noted that for $M \to 1$, $S \to 0$ and $G \to 0$, the above dispersion relation becomes the dispersion relation in the open water region in a two-layer fluid. The unknown coefficients can be obtained by solving $2N_0 + 5$ equations from the matching and edge conditions, where N_0 is the truncation value of the series after which the above relations converge. After obtaining the unknown coefficients, vertical wave force exerted on the membrane, deflection of the membrane and flow distribution around the membrane are evaluated and analyzed for various wave and structural parameters.

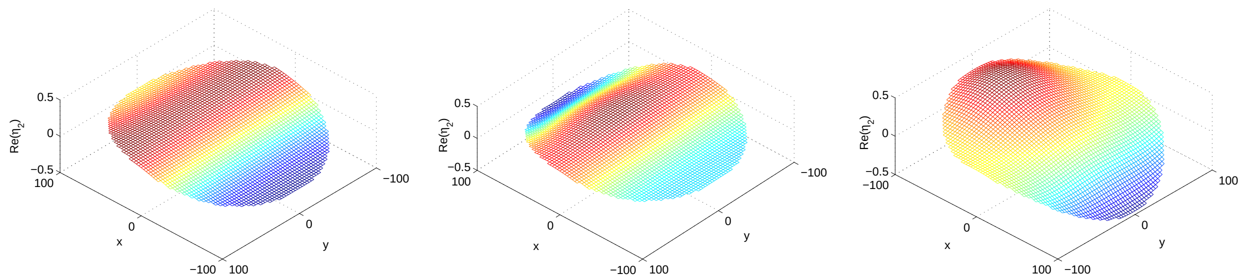
4 Results and discussion

In the numerical computation, values of different physical parameters that are assumed to be fixed are: water depth $H = 50\,\mathrm{m}$, radius of the membrane $a/H = 2$, density of water $\rho_2 = 1025\,\mathrm{kg/m^3}$, non-dimensional tensile force $T/\rho_2 gH^2 = 0.1$ and non-dimensional thickness $d/H = 0.01$. The truncated value of $M_0 = 8$ and $N_0 = 8$. The vertical wave force $\mathbb{C}_V$ acting on the porous membrane is given as $\mathbb{C}_V = \mathrm{i}\omega\rho\int_0^{2\pi}\int_0^a \phi_2(r,\theta,0)r\,dr\,d\theta$. The non-dimensional form is given by $F_V = |\mathbb{C}_V|/(\rho gH^3)$. In Fig. 2, the vertical wave force exerted on the membrane is plotted as a function of the non-dimensional wavenumber for different values of (a) porous-effect parameter G, (b) depth ratio h/H and (c) density ratio s. From Fig. 2(a), it is observed that due to wave energy dissipation by the porous membrane, the vertical force on the porous membrane decreases with an increase in the absolute value of the porous-effect parameter G. Fig. 2(b) reveals that the wave force on the structure is less when the interface is closer to the free surface which is due to the more energy concentration near the free surface. From Fig. 2(c), it is seen that for decrease in the values of s, the pressure exerted on the membrane more on the upper layer due to the fact that $\rho_1 < \rho_2$. Thus, the vertical force exerted on the membrane increases for decrease in the value of s.



(a) (b) (c)

Figure 2: Vertical force F_V against the non-dimensional wavenumber $k_I H$ for different values of the (a) porous-effect parameter G with $h/H = 0.5$ and $s = 0.8$, (b) depth ratio h/H with $s = 0.8$ and $G = 0.25 + 0.25\mathrm{i}$, and (c) density ratio s with $h/H = 0.5$ and $G = 0.25 + 0.25\mathrm{i}$.



(a) $G = 0$ (b) $G = 0.25 + 0.25\mathrm{i}$ (c) $G = 0.5 + 0.5\mathrm{i}$

Figure 3: Real part of deflection of the porous membrane for different values of the porous-effect parameter G with $h/H = 0.5$ and $s = 0.8$.

The deflection of the circular porous membrane is plotted in Fig. 3 for different values of the porous-effect parameter G. It is found that for higher absolute values of G, the wave energy dissipation increases, thus the wave force exerted on the membrane decreases as shown in Fig. 2(a). Therefore, the membrane deformation decreases, and it is inferred that less transmission occurs in the lee side of the membrane and a significant reduction in amplitude of the surface and interface elevations.

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