The best of both worlds: integrating psychological and econometric theories of choice

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Collaborators

UTS & CenSoC

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Overview

- Random utility models
  - Limitations
- Perceptual choice models
  - Evidence accumulation
  - Strengths
  - Limitations
- Integrating the fields
  - Prior work
  - Limitations
  - New approaches

Discrete Choice Experiments

Which one do you prefer?

Utility

Utility = 3.5

Utility = 0.2

Utility = 2.5

Utility = 3.1

Random Utility Models

Utility = 3.5

Utility = 0.2

Utility = 2.5

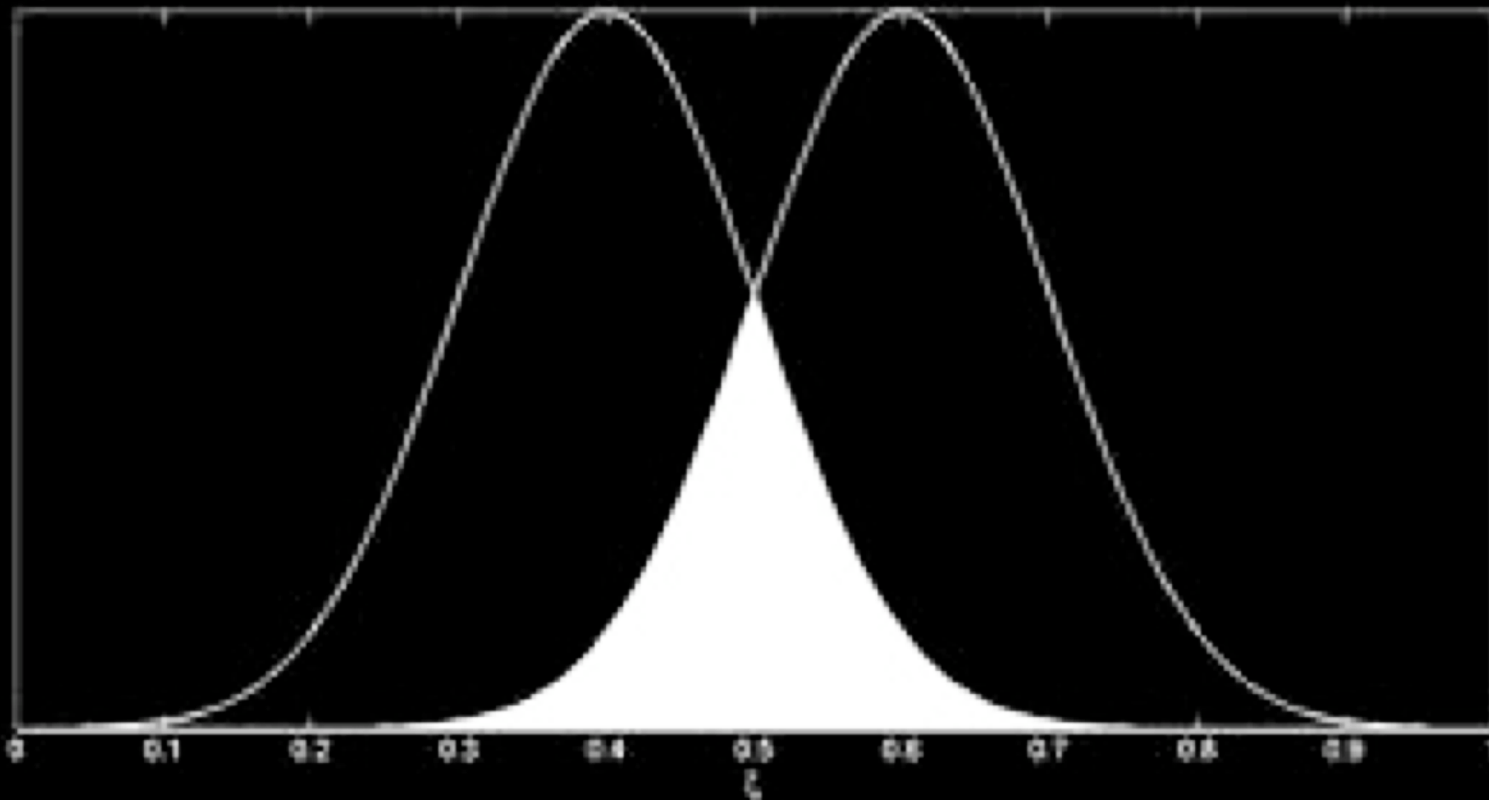
Utility = 3.1

Utility = 3.3

Utility = 0.5

Utility = 1.6

Utility = 3.4



$$\frac{\exp\left[\lambda \sum_{k=1}^{K} \beta_k X_{kj}\right]}{\sum_{j=1}^{N} \exp\left[\lambda \sum_{k=1}^{K} \beta_k X_{kj}\right]}$$

Random Utility Model

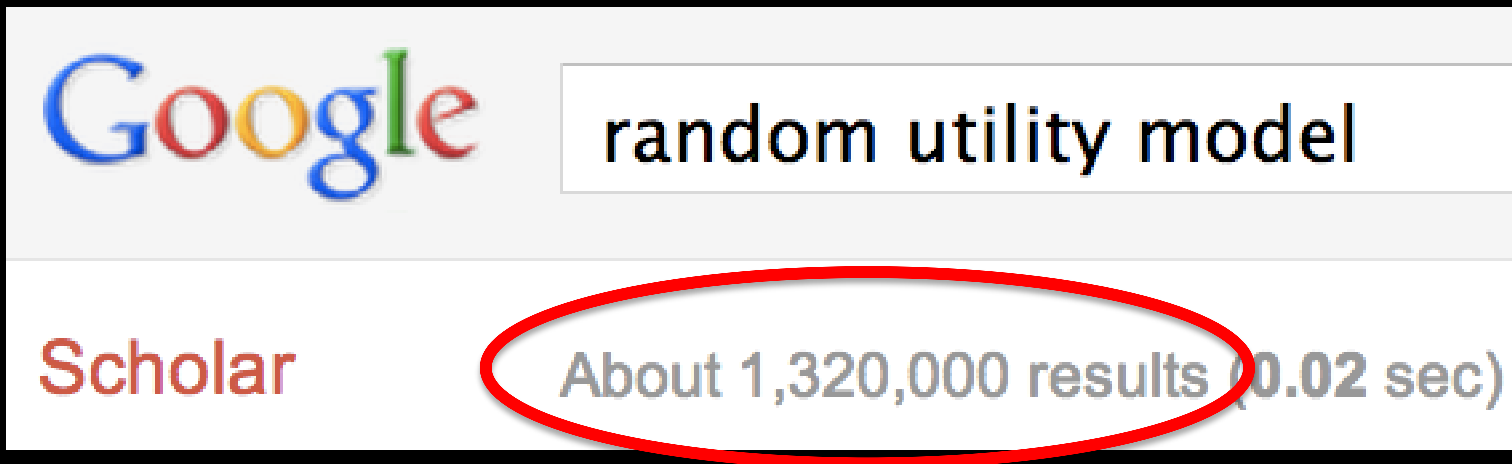
What we *get*

- Observed choices

What we *want*

- Willingness to buy
- Preference strength
- “utility”

Advantages of Random Utility Models



Statistical Inference

BMC Medical Research Methodology
BioM
Research article
Open

Estimating preferences for a dermatology consultation using Best-Worst Scaling: Comparison of various methods of analysis

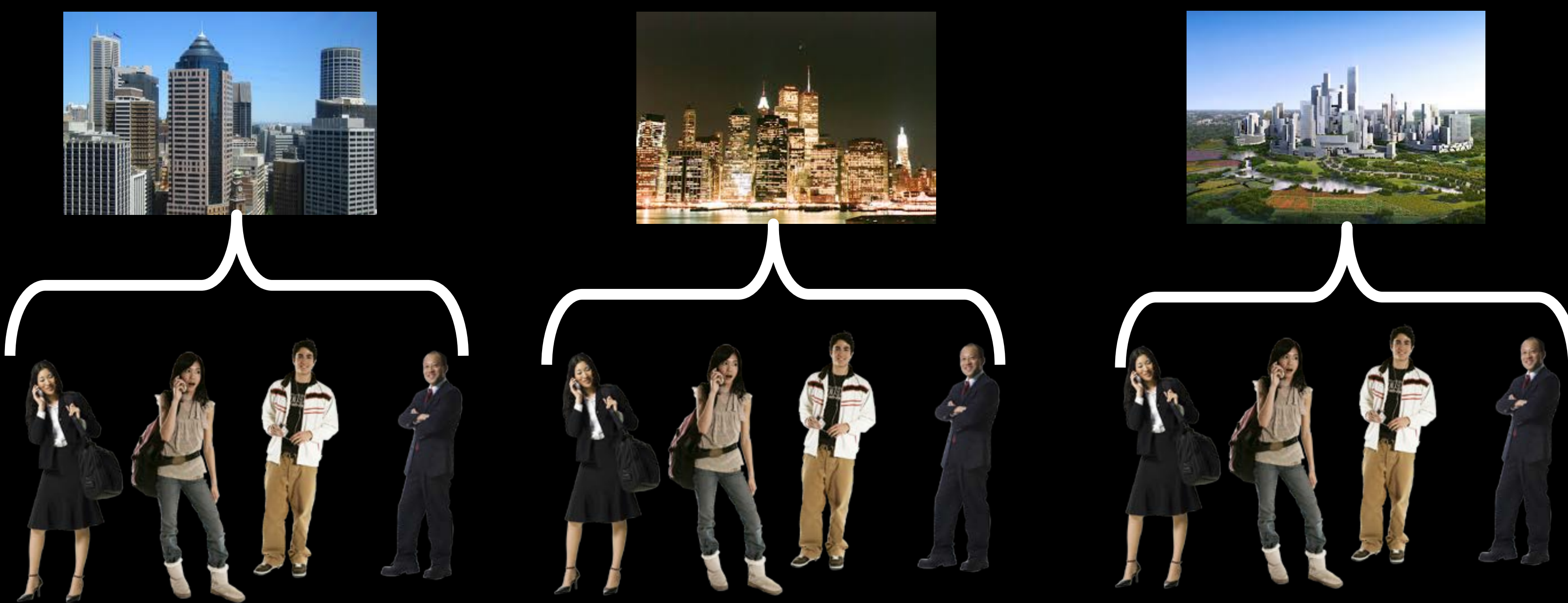
Terry N Flynn[1], Jordan J Louviere[2], Tim J Peters[3] and Joanna Coast*[4]

Table 2: Paired model conditional logit estimates

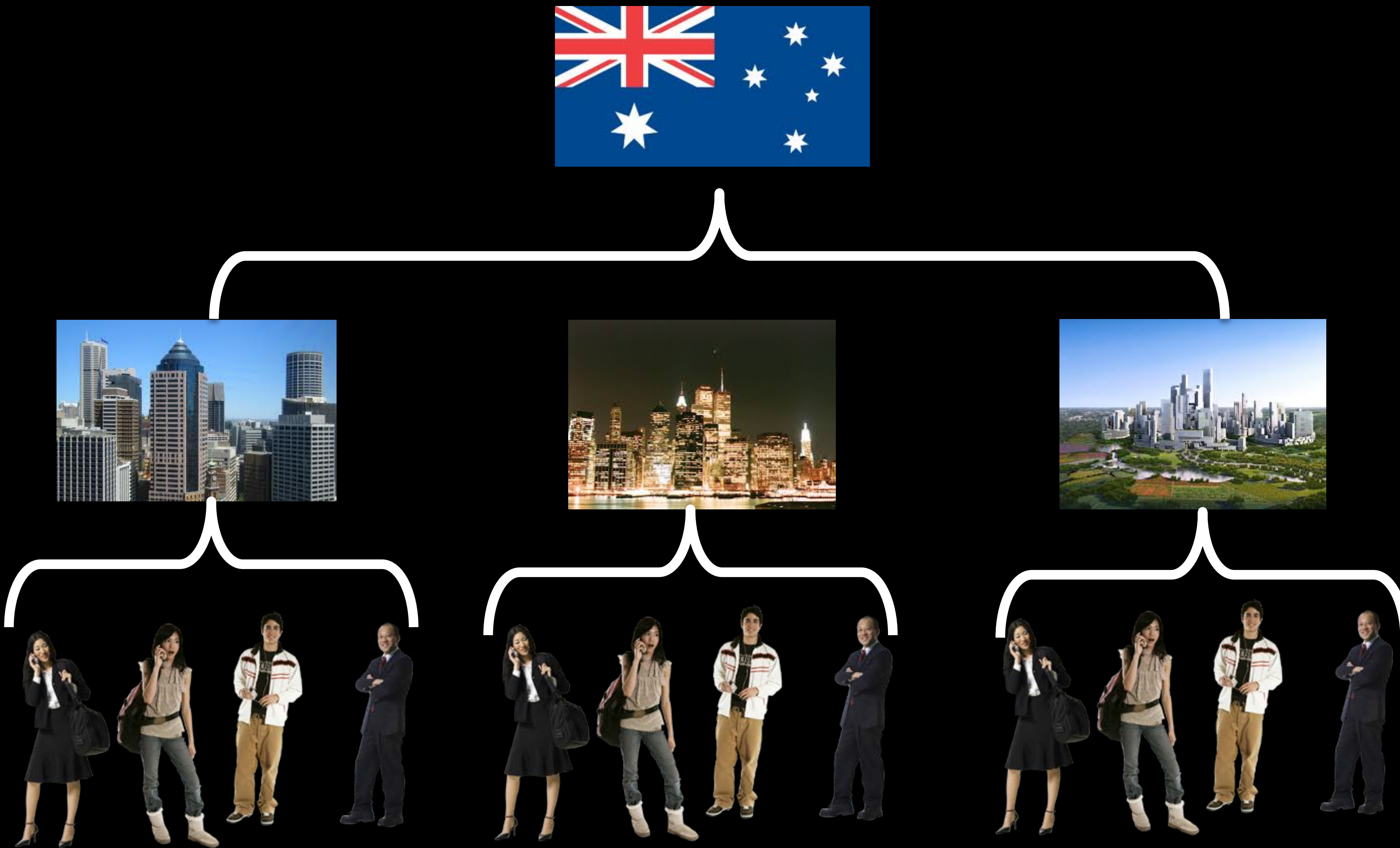
	Coefficient	Standard error	95% Confidence interval	
Attribute impacts				
Waiting time	-	-	-	-
Doctor	1.3687	0.2011	0.9745	1.7628
Convenience	0.5060	0.1182	0.2743	0.7377
Thoroughness	0.3710	0.1358	0.1048	0.6372

Sophisticated Statistical Frameworks

Sophisticated Statistical Frameworks



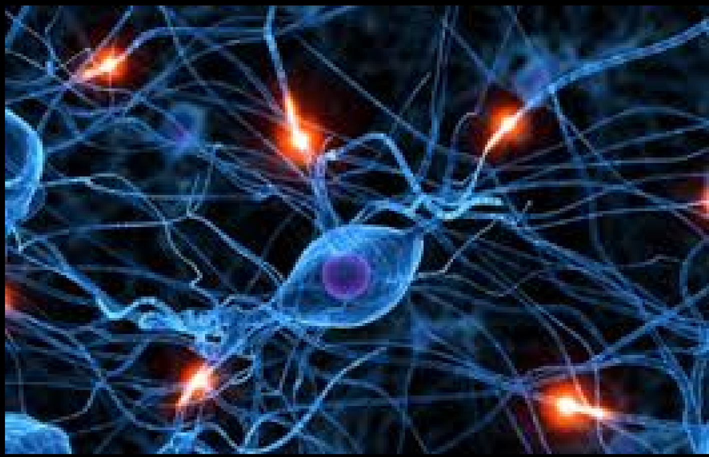
Sophisticated Statistical Frameworks



Limitations of
Random Utility Models

Limitations of Random Utility Models

?



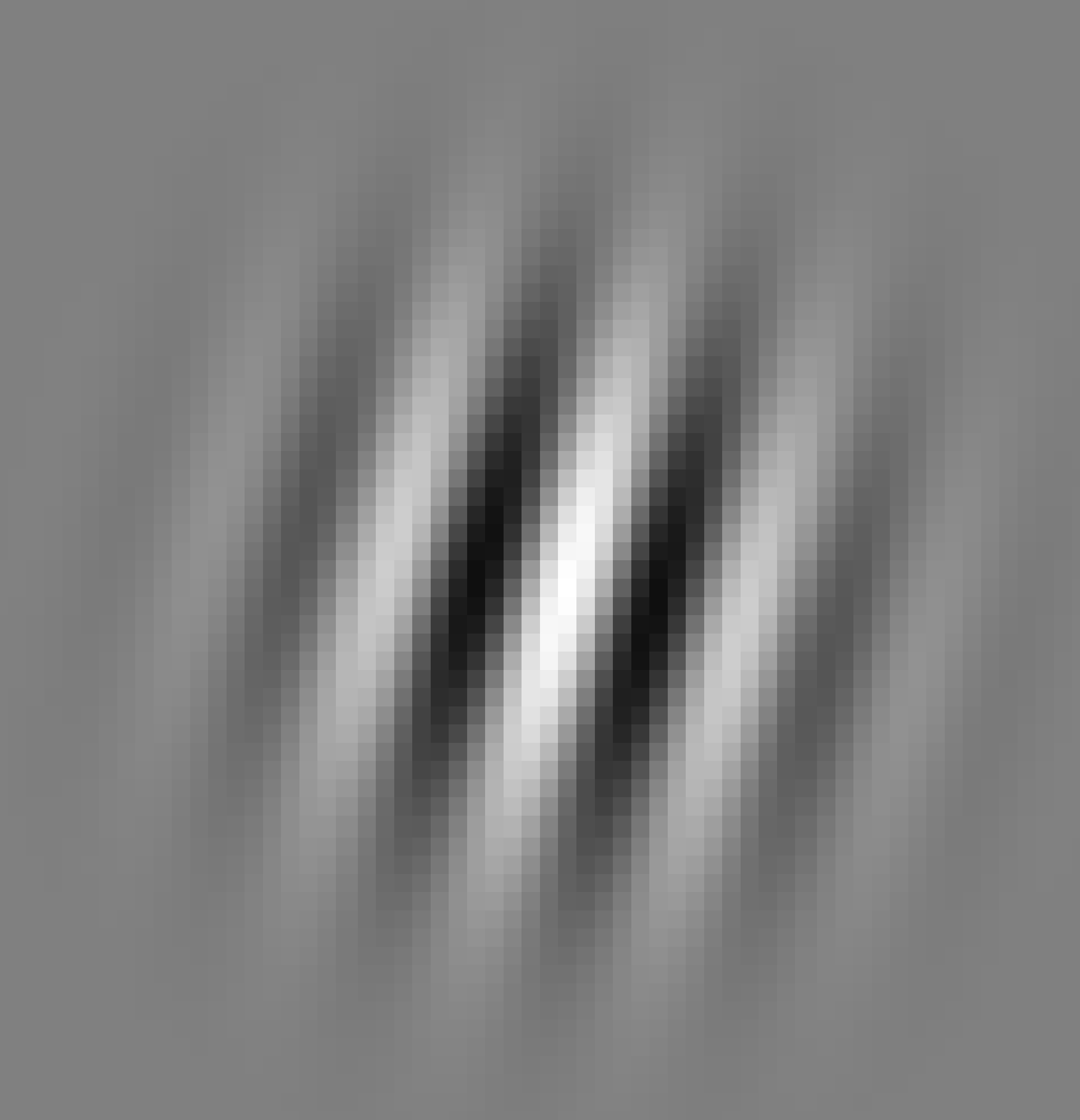
Limitations of Random Utility Models

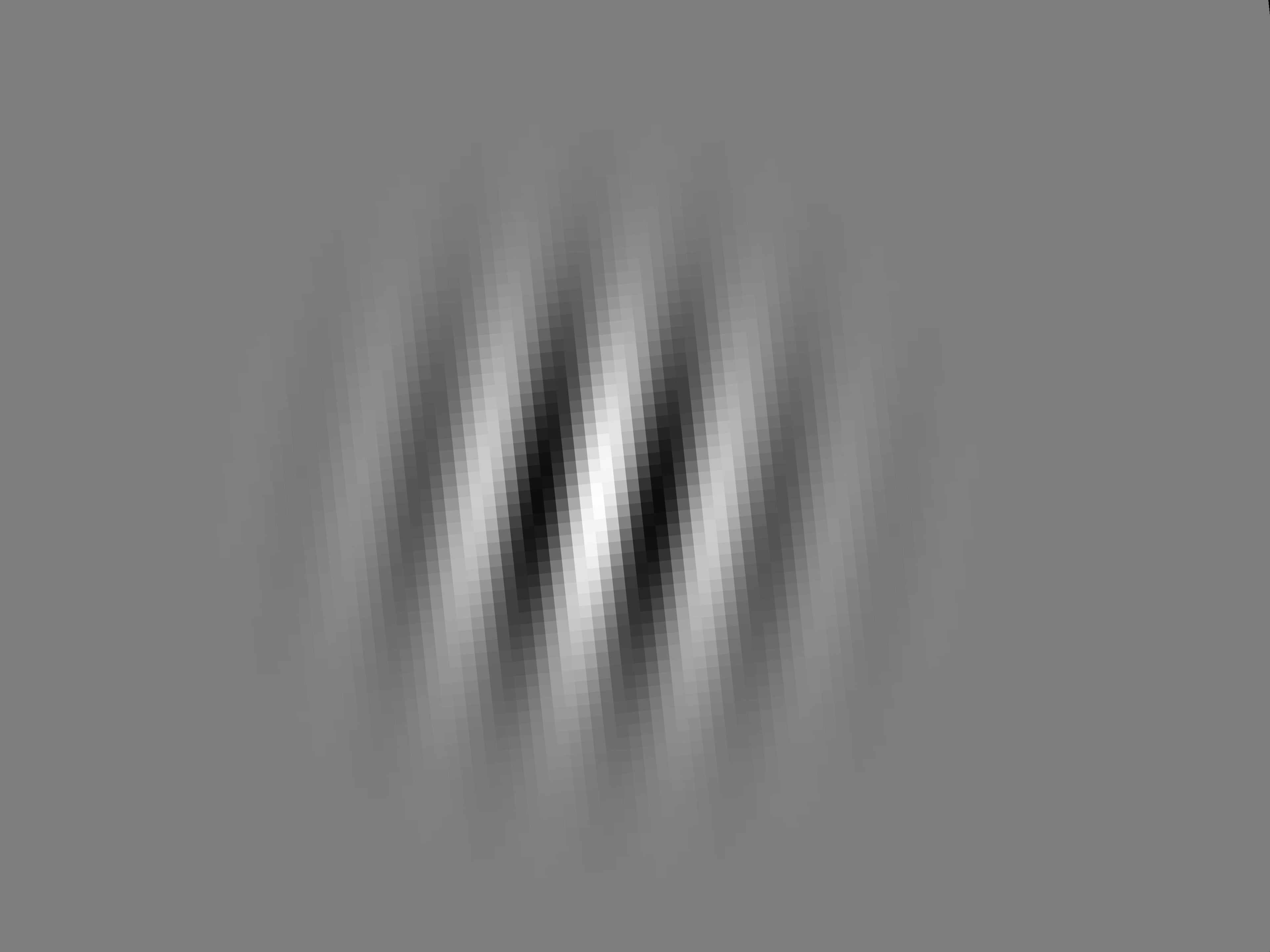
?

Limitations of Random Utility Models

?

Perceptual Choice





Loudness

Pitch

Brightness

Motion direction

....

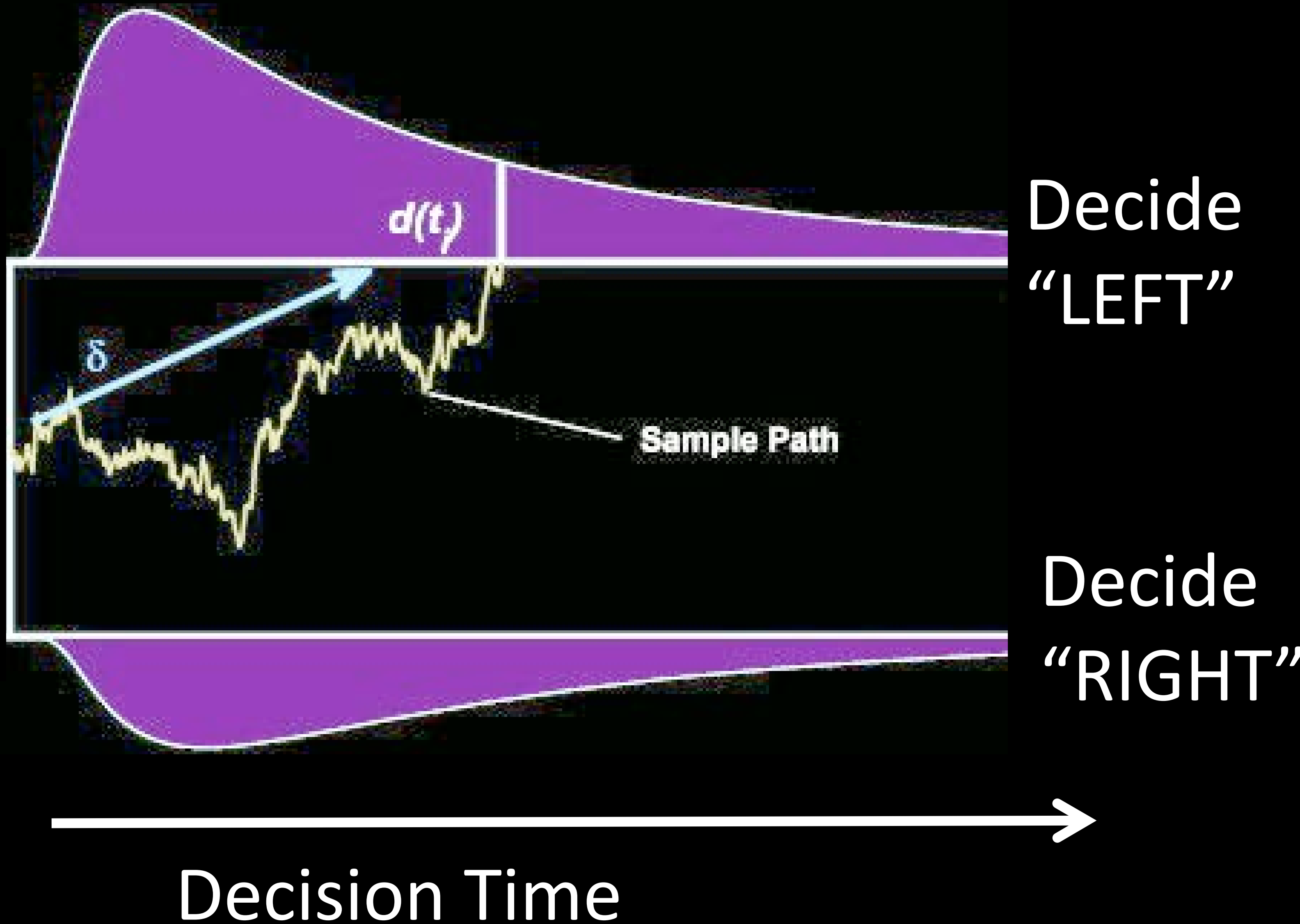
Not just perception:

Lexical processing
Short term memory
Simple detection
....

Small sample = 3 people
Large sample = 100 people

Small sample = 3 people
Large sample = 100 people

Few decisions = 100 trials
Many decision = 10,000 trials


d(t)
δ
Sample Path
Decide
“LEFT”
Decide
“RIGHT”
Decision Time

Stone (1960): Random walk

Laming (1968): Random walk + variance in initial evidence

Vickers (1970): Accumulator model

Ratcliff (1978): Random walk + variance in drift rate

Ratcliff & Rouder (1998): RW with variance in drift and initial evidence

Smith & van Zandt (2000): Time-varying accumulator model

Usher & McClelland (2001): Leaky, competing accumulator model

Ratcliff & Teurlinckx (2002): RW with the lot

Brown & Heathcote (2005,2008): Ballistic accumulators

Wagenmakers et al. (2007): Simplified random walk (EZ)

Evidence Accumulation Model

What we *get*

- Observed choices
- Response times
- Possibly neural measurements too

What we *want*

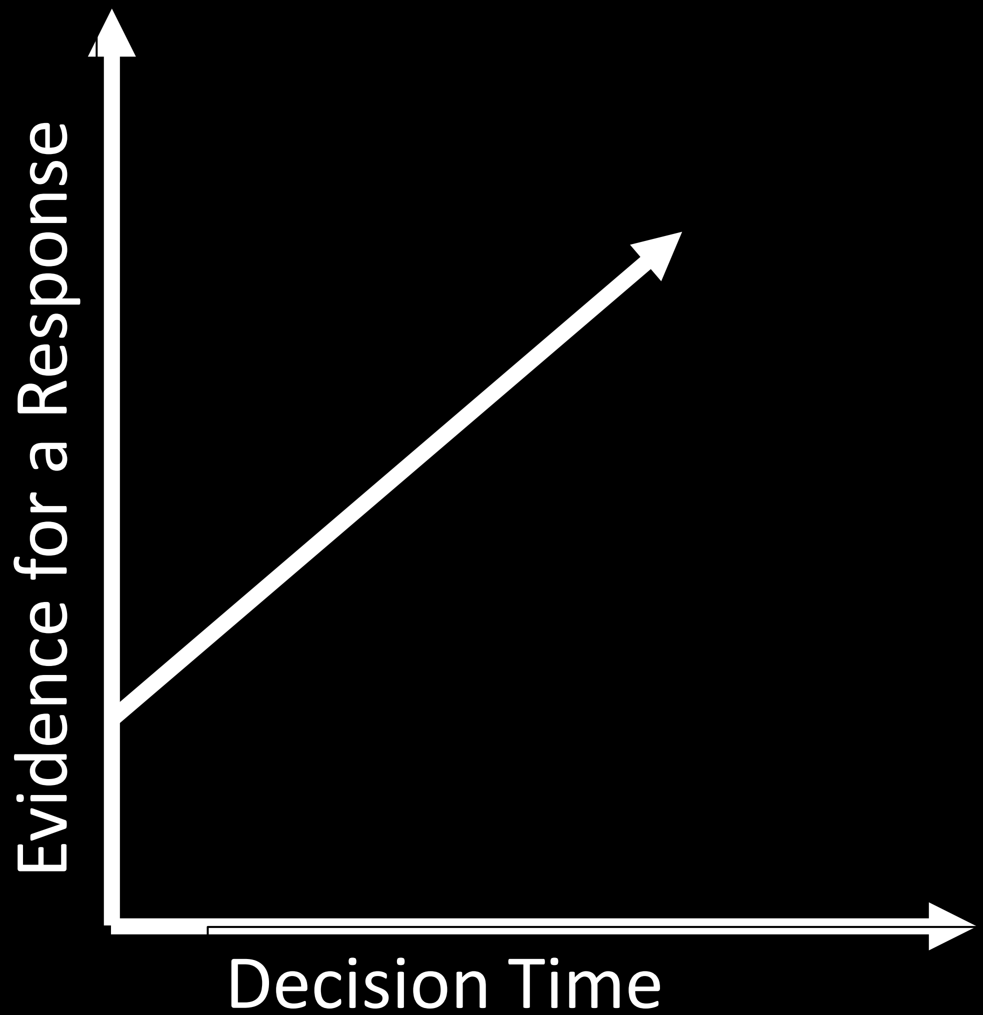
- Latent cognitive processing
- How are decisions made?
- What influences them?
- What causes variability?

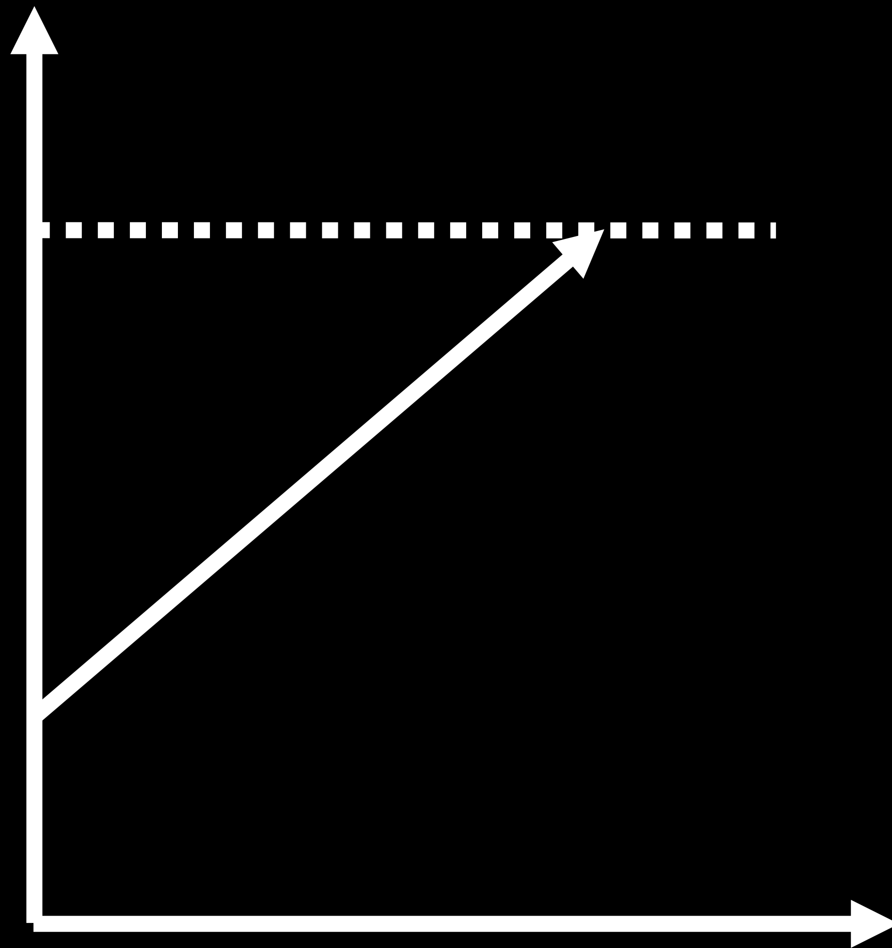
Linear Ballistic Accumulator Model

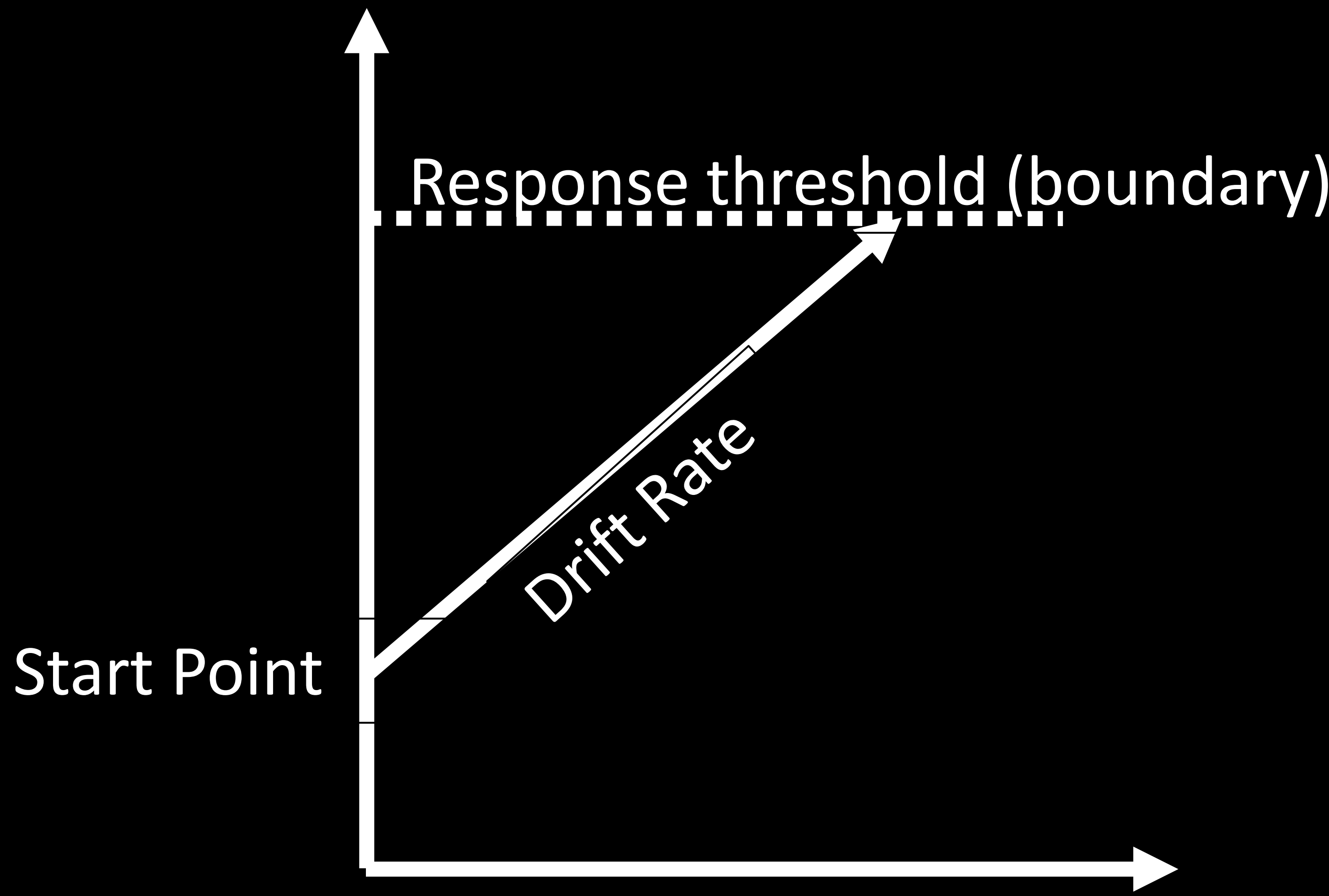
LBA: Brown & Heathcote, 2008, *Cognitive Psychology*

Turner, Sederberg, Brown, & Steyvers, 2013, A Note on Efficiently Sampling from Distributions with Correlated Dimensions. *Psychological Methods*.

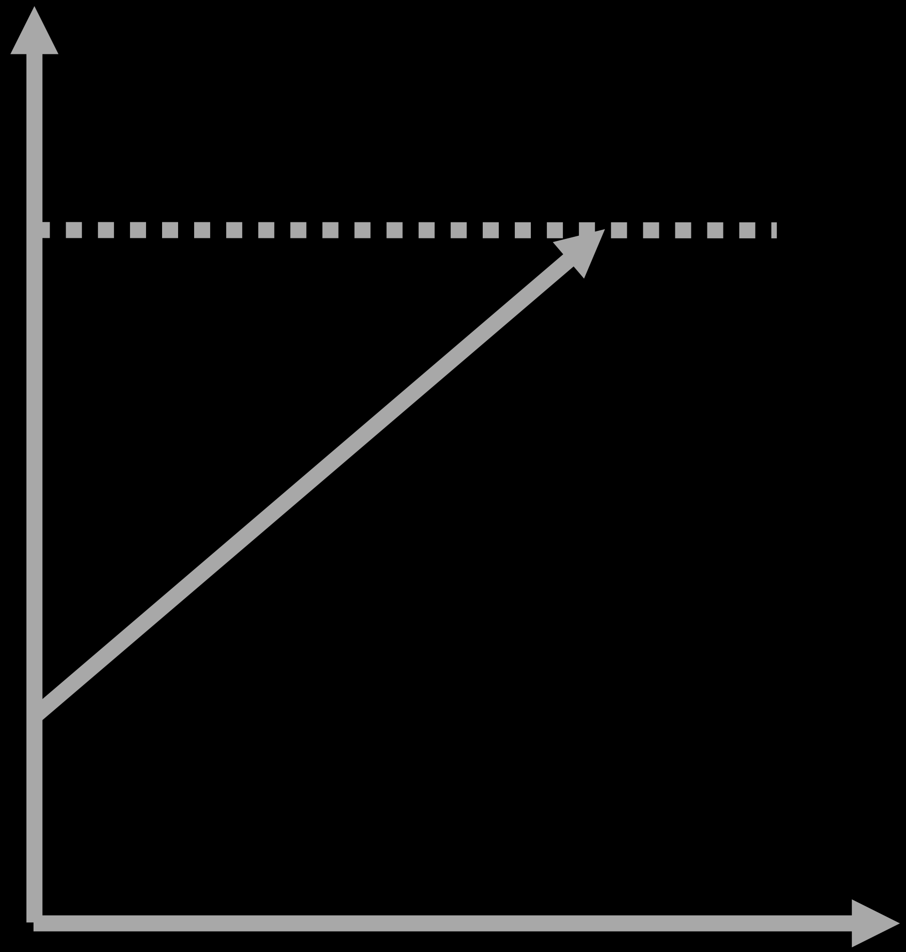
Donkin, Brown, & Heathcote, 2011, Drawing conclusions from choice response time models: a tutorial using the LBA. *Journal of Mathematical Psychology*.


Evidence for a Response
Decision Time

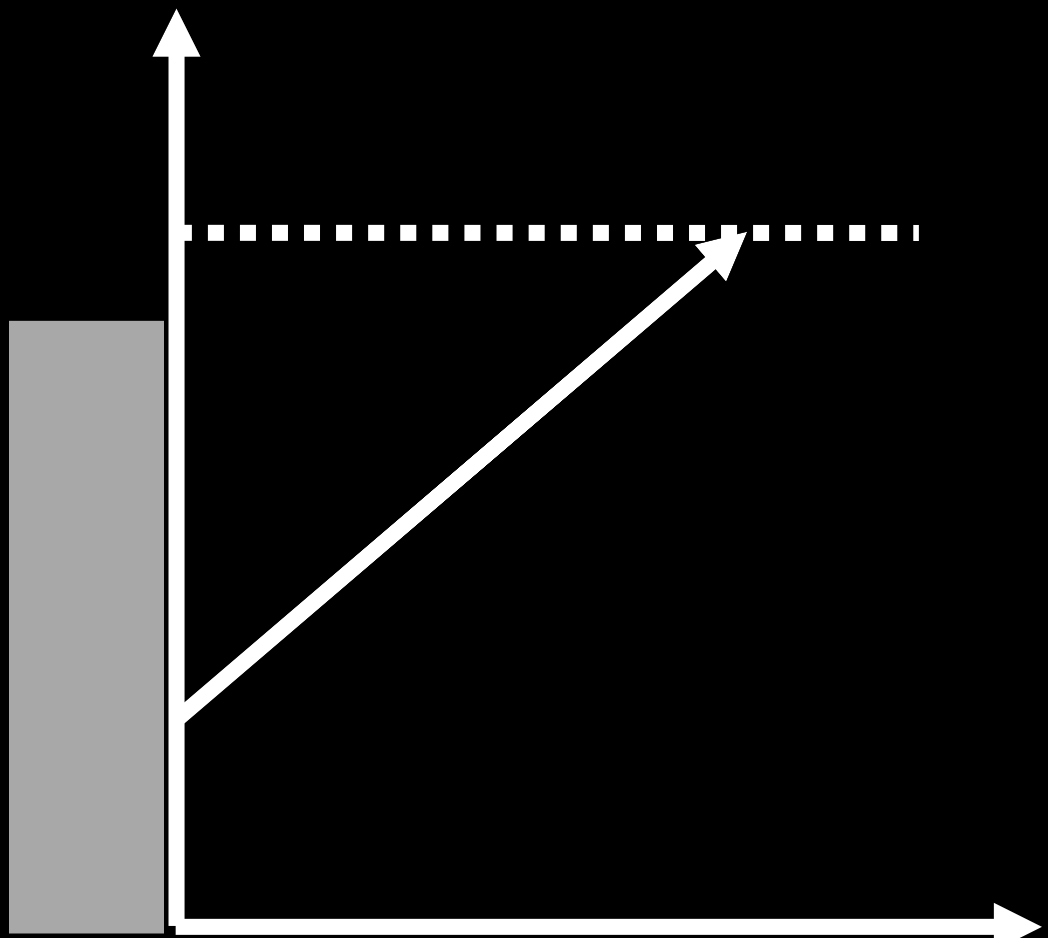


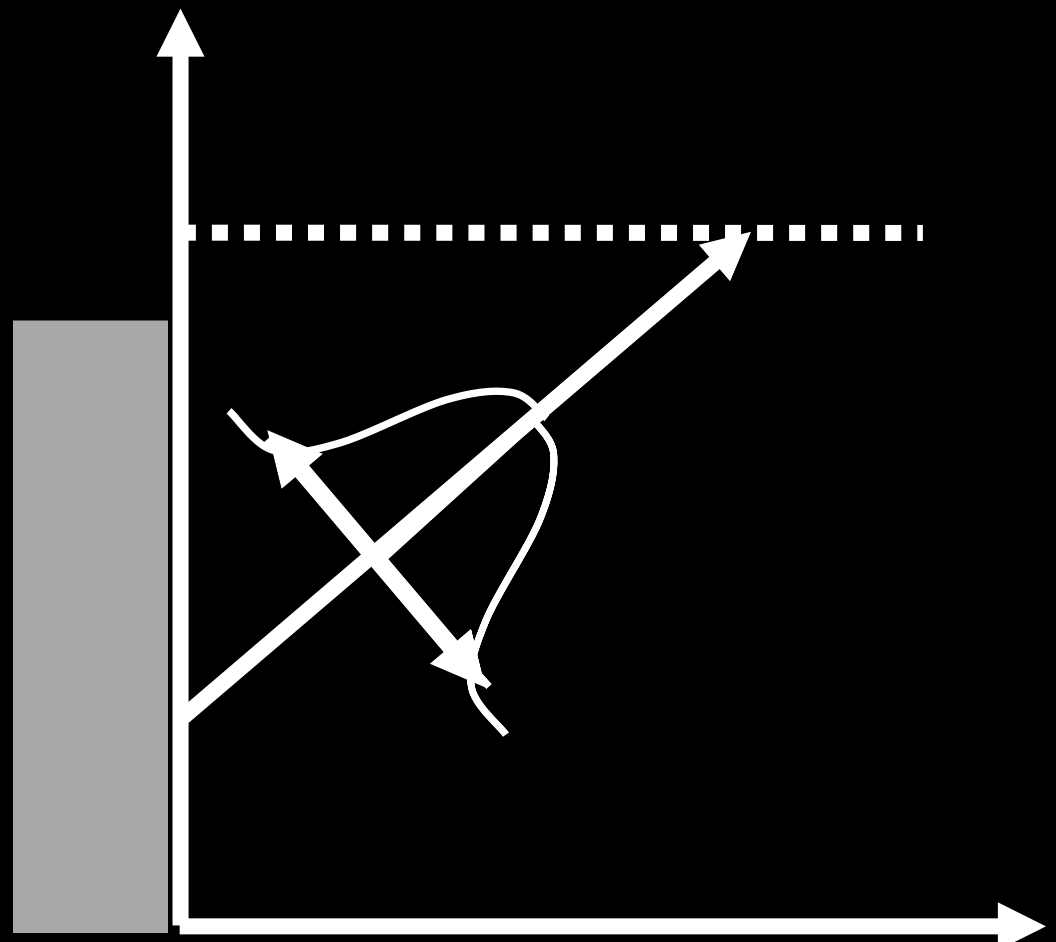


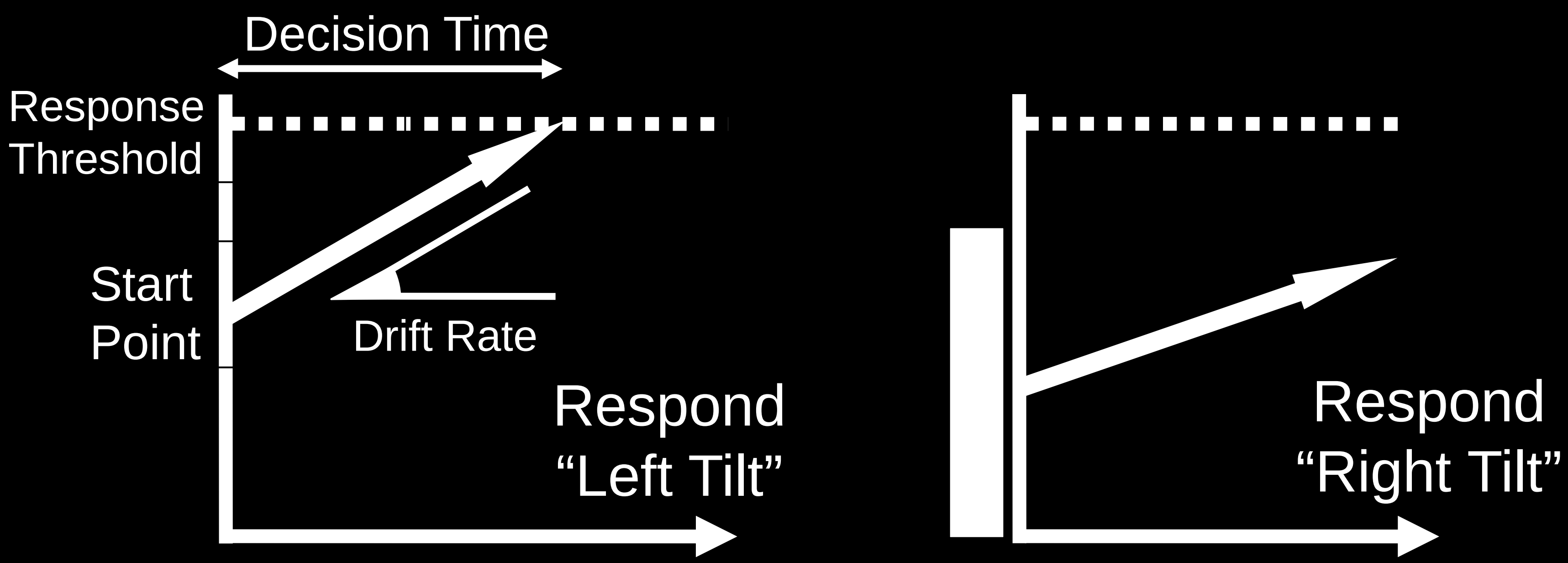
Response threshold (boundary)
Drift Rate
Start Point



RT = (threshold – start) ÷ (drift)






Decision Time
Response
Threshold
Start
Point
Drift Rate
Respond
“Left Tilt”
Respond
“Right Tilt”

The CDF for first passage times on a single accumulator is:

$$F_i(t) = 1 + \frac{b - A - tv_i}{A}\Phi\left(\frac{b - A - tv_i}{ts}\right) - \frac{b - tv_i}{A}\Phi\left(\frac{b - tv_i}{ts}\right)$$
$$+ \frac{ts}{A}\phi\left(\frac{b - A - tv_i}{ts}\right) - \frac{ts}{A}\phi\left(\frac{b - tv_i}{ts}\right)$$

The associated PDF is:

$$f_i(t) = \frac{1}{A}\left[-v_i\Phi\left(\frac{b - A - tv_i}{ts}\right) + s\phi\left(\frac{b - A - tv_i}{ts}\right) + v_i\Phi\left(\frac{b - tv_i}{ts}\right) - s\phi\left(\frac{b - tv_i}{ts}\right)\right]$$

The CDF for first passage times on a single accumulator is:

$$F_i(t) = 1 + \frac{b-A-tv_i}{A}\Phi\left(\frac{b-A-tv_i}{ts}\right) - \frac{b-tv_i}{A}\Phi\left(\frac{b-tv_i}{ts}\right) + \frac{ts}{A}\phi\left(\frac{b-A-tv_i}{ts}\right) - \frac{ts}{A}\phi\left(\frac{b-tv_i}{ts}\right)$$

The associated PDF is:

$$f_i(t) = \frac{1}{A}\left[-v_i\Phi\left(\frac{b-A-tv_i}{ts}\right) + s\phi\left(\frac{b-A-tv_i}{ts}\right) + v_i\Phi\left(\frac{b-tv_i}{ts}\right) - s\phi\left(\frac{b-tv_i}{ts}\right)\right]$$

For any number of choice alternatives, joint density is:

$$\mathrm{PDF}_i(t) = f_i(t)\prod_{j\neq i}\left(1 - F_j(t)\right)$$

$$\mathrm{PDF}_i(t) = f_i(t) \prod_{j \neq i} \left(1 - F_j(t)\right)$$

The probability that choice *j* has not reached threshold by time *t*.

The density function for choice *i* reaching threshold at time *t*.

Open Source Software – Multi-platform, multi-language, flexible and general.

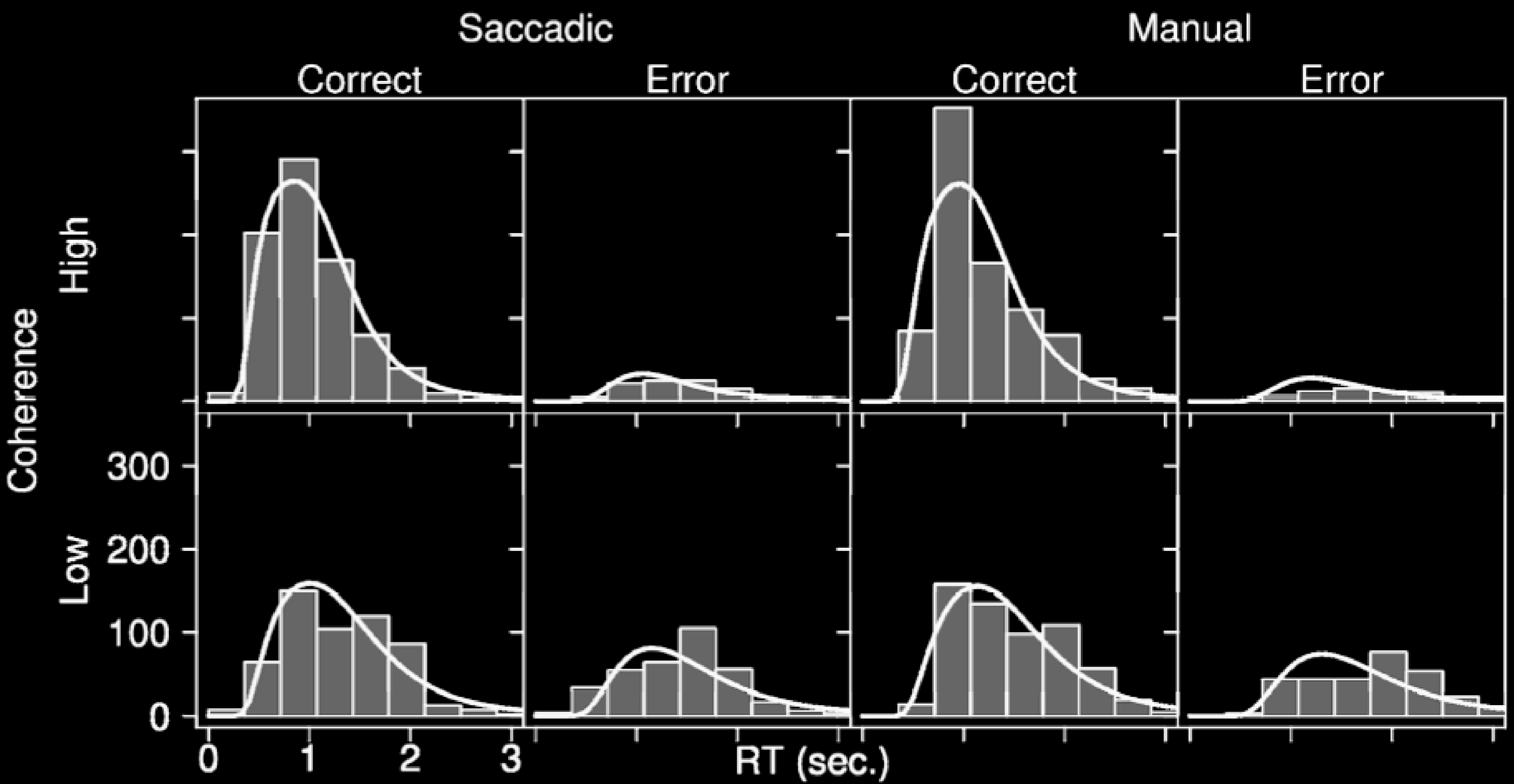
http://newcl.org/Brown

R, Matlab, MS Excel, Python.

Strengths of Perceptual Choice Models

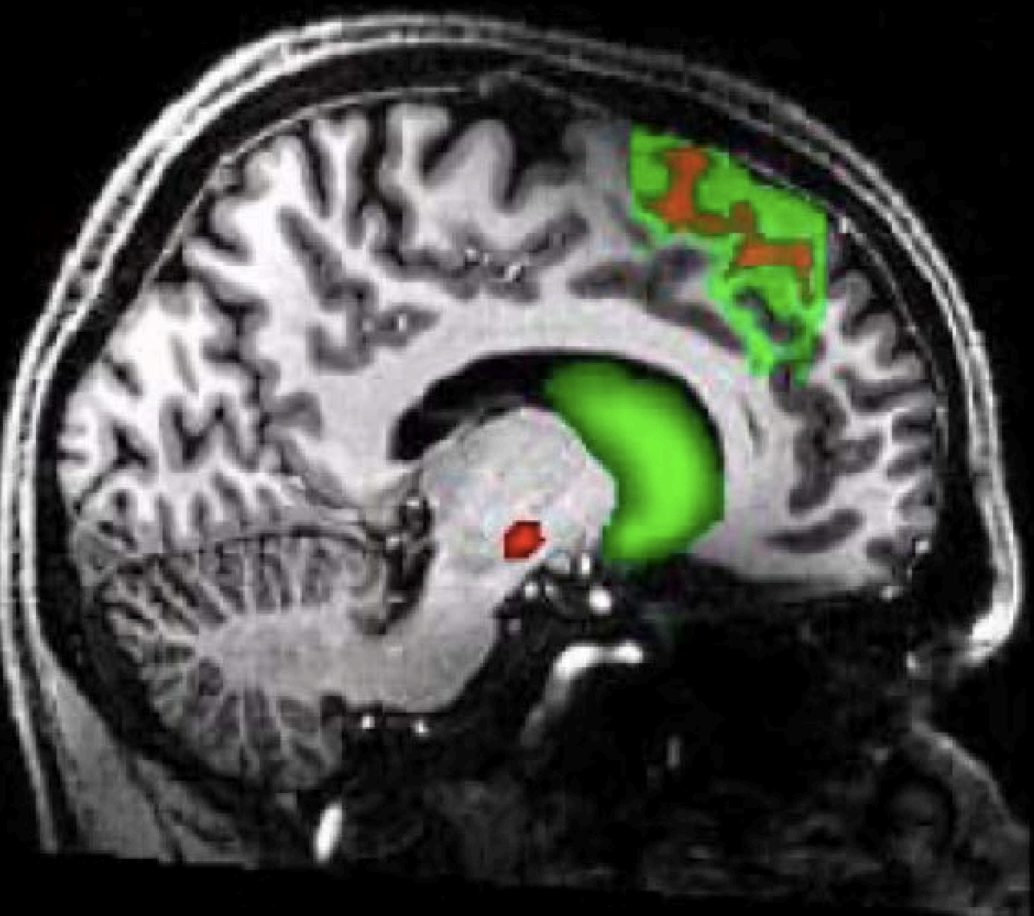
- Predict and understand response times
  - These are becoming ubiquitous, and often wasted
- Detailed and carefully grounded neurophysiological links
  - Structural *and* functional
- A cognitive process-level account
  - E.g. balancing speedy vs. careful decisions
- Access to some otherwise-difficult quantities
  - E.g. variance parameters, timing parameters

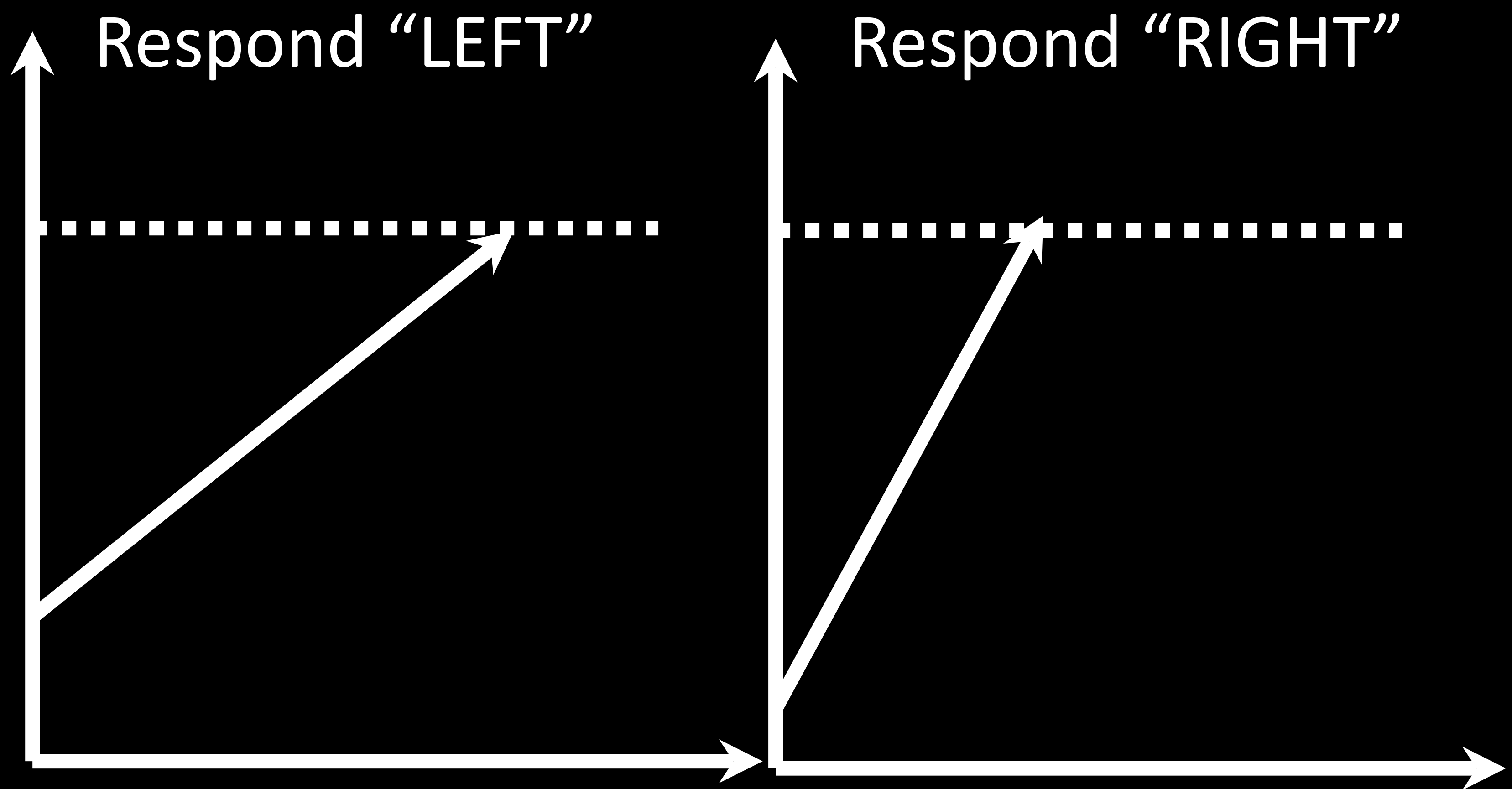
Response Time



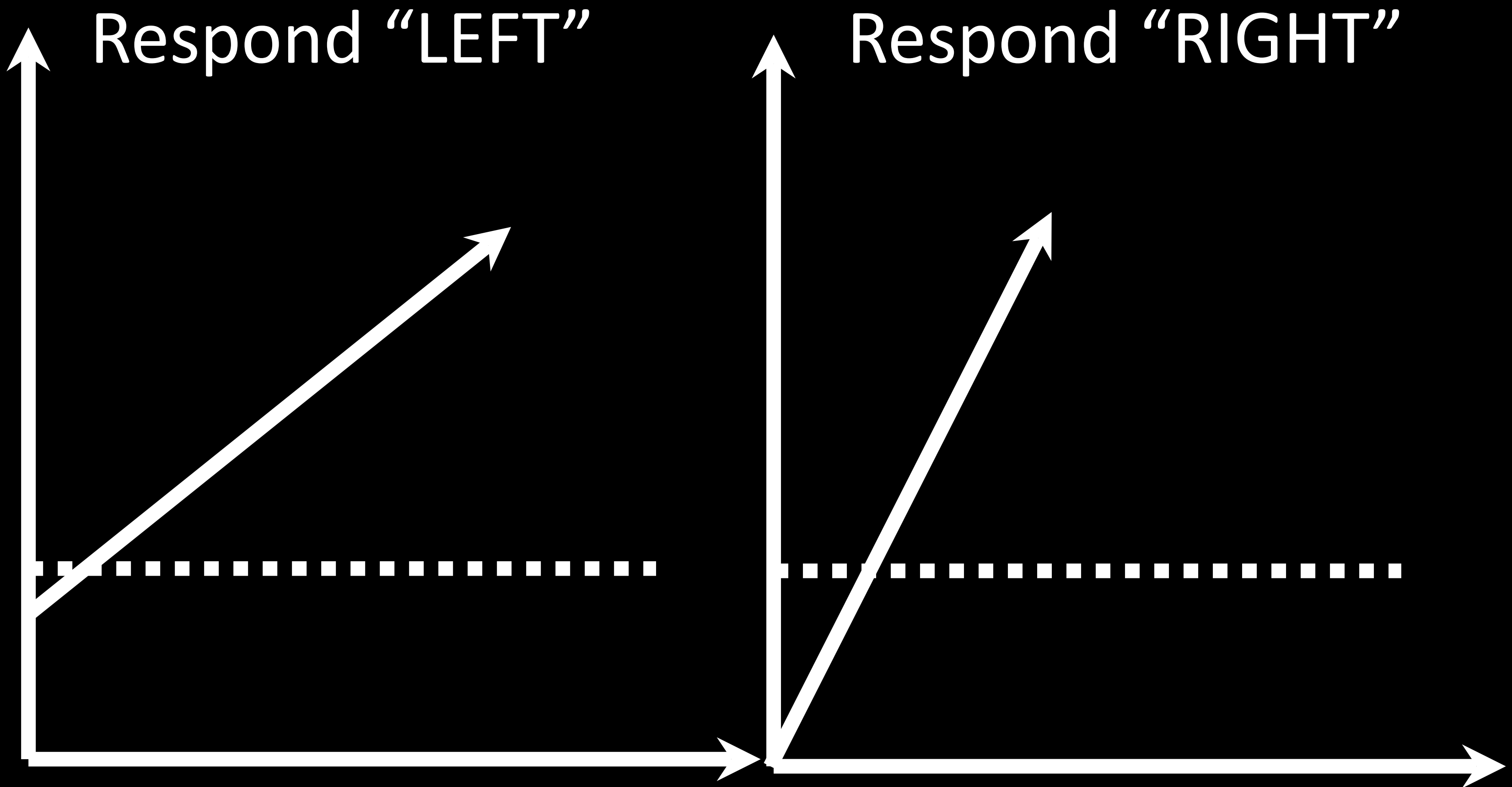
Speed-Accuracy Tradeoff

- Ubiquitous
- Important
- Neural basis well understood
- Confounds experiments!

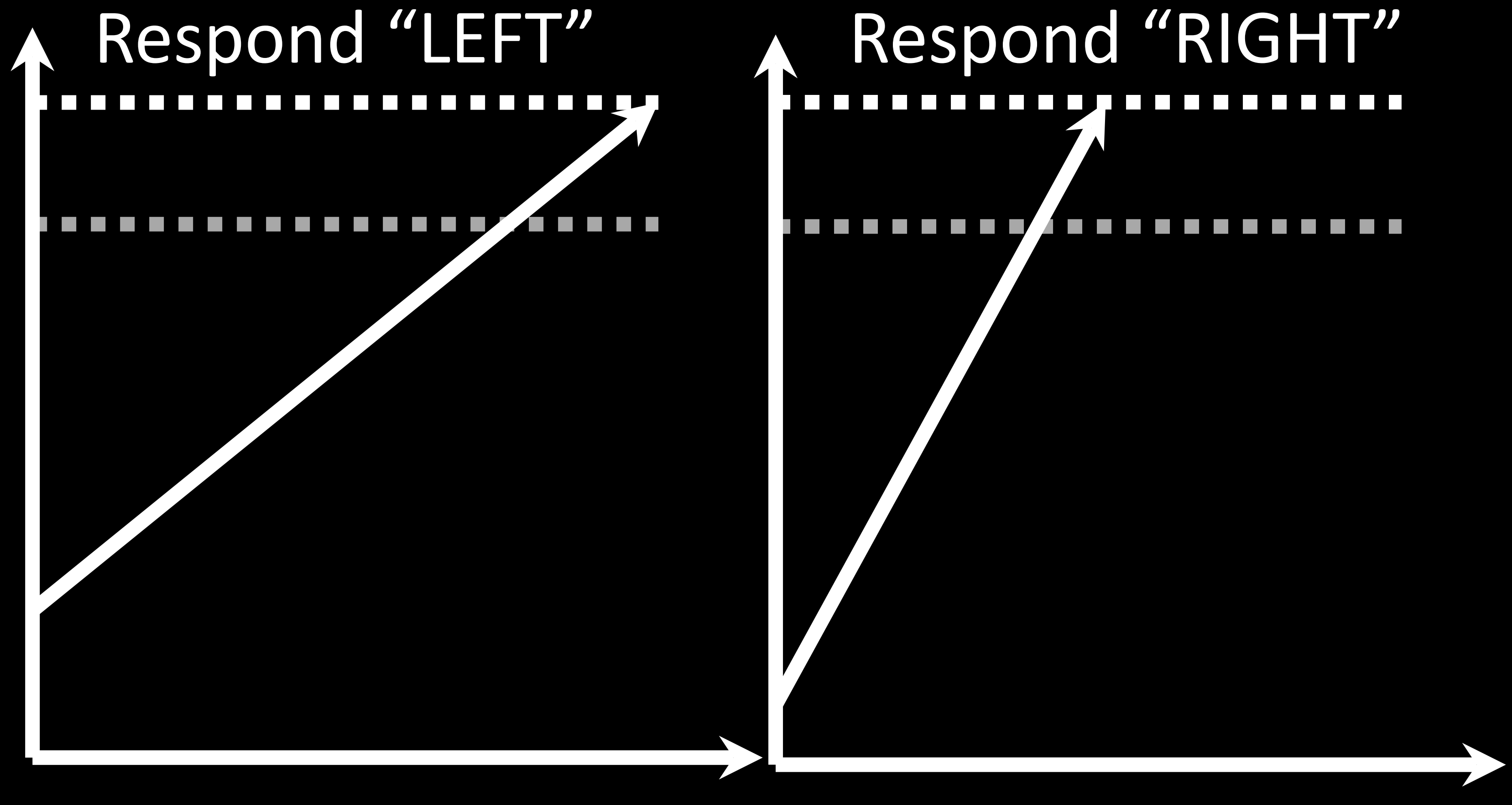



Respond “LEFT”
Respond “RIGHT”

Faster, but less carefully



Slower and more carefully



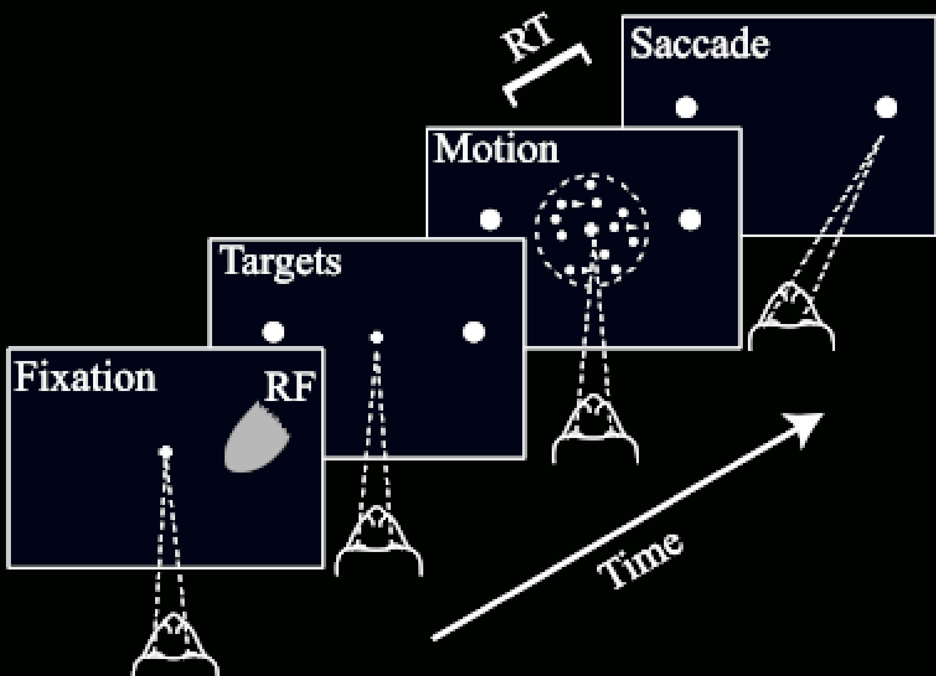
Detailed Neural Links

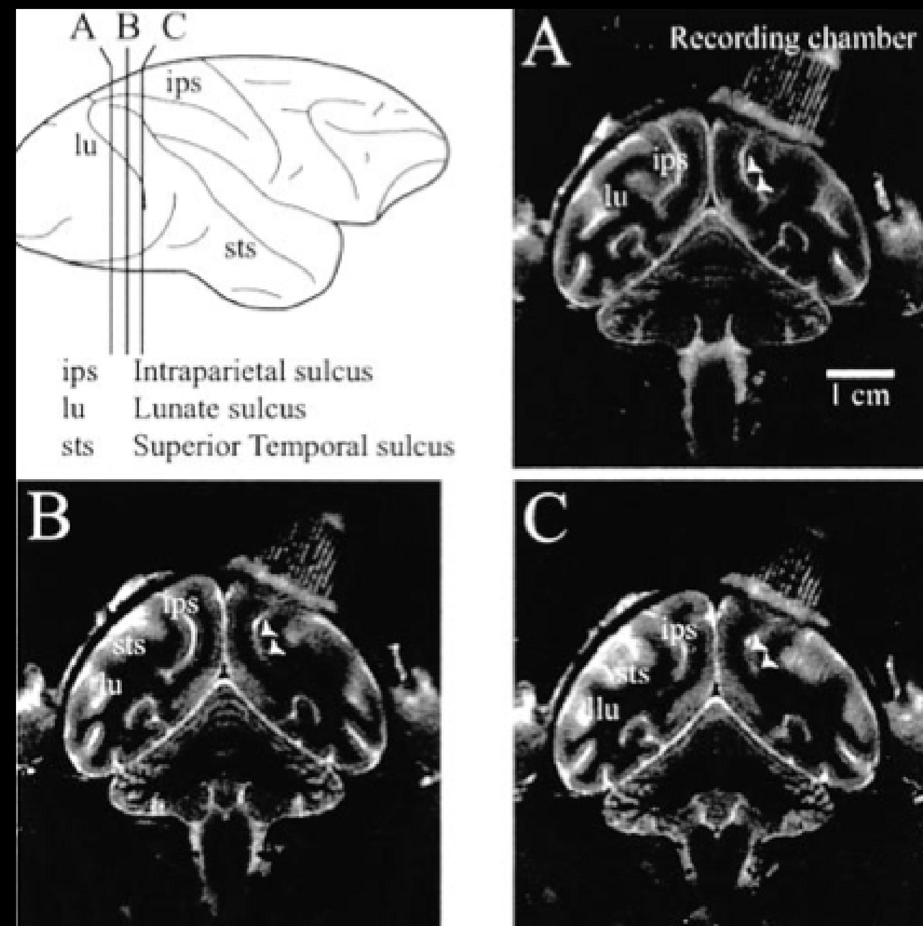
- Shadlen, Newsome, Britten, et al.
- Schall, Palmeri, Logan, et al.
- Forstmann, Wagenmakers, et al.
- Serences, Boynton, et al.
- Bogacz, McClelland, Usher, et al.

The Journal of Neuroscience, November 1, 2002, 22(21):9475–9489

Response of Neurons in the Lateral Intraparietal Area during a Combined Visual Discrimination Reaction Time Task

Jamie D. Roitman[1] and Michael N. Shadlen[2]





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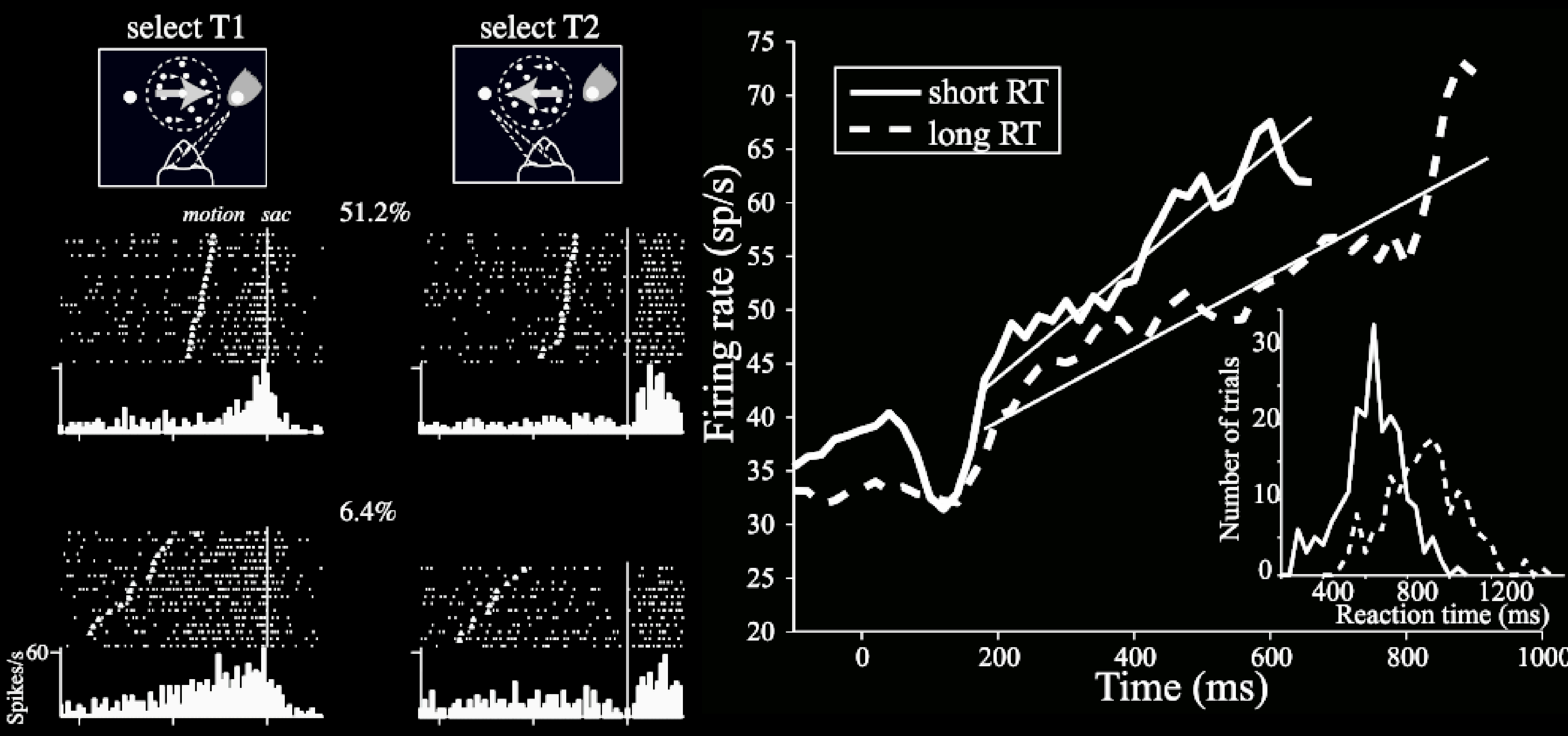

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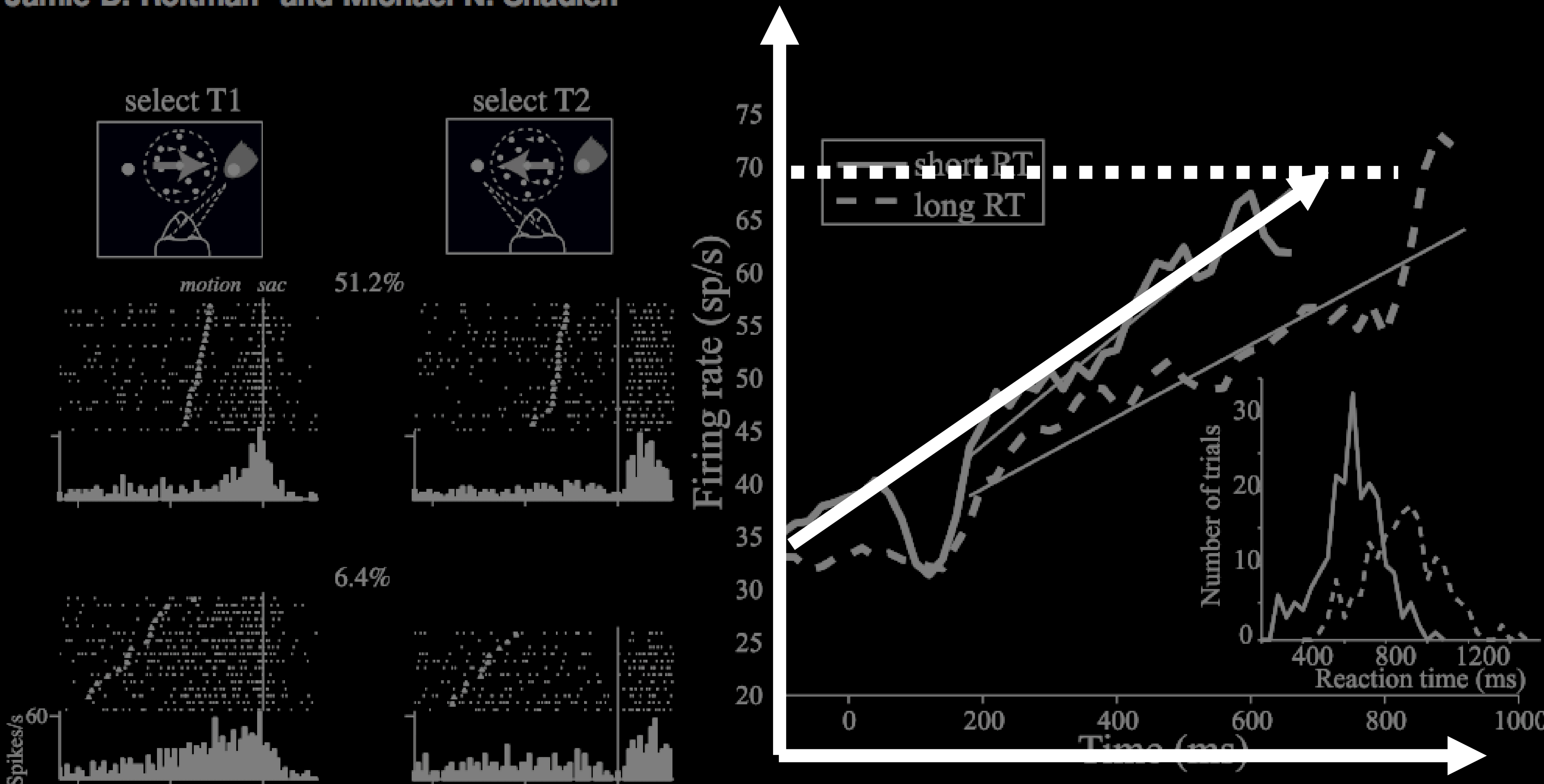
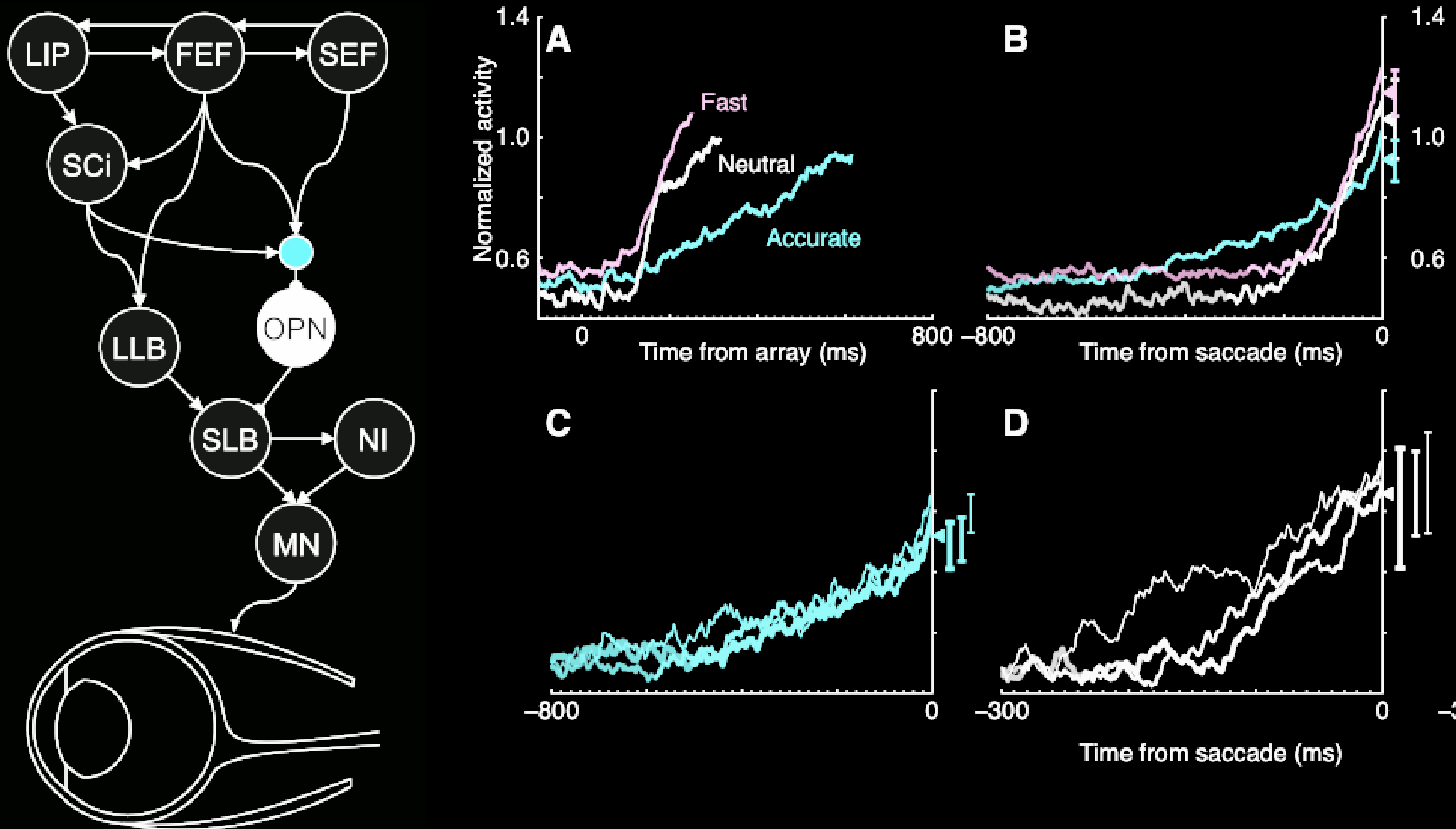




Neural Mechanisms of Speed-Accuracy Tradeoff

Richard P. Heitz and Jeffrey D. Schall

In humans, that would be done without the needles & surgery.

Typical LBA Example with Perceptual Choice (no surgery!)

Tiffany Ho, John Serences (UCSD)

Myself and Pete Cassey (U. Newcastle)

Major Depressive Disorder

Decision 1
Decision 2
....
Decision 60

Yoked Control Sample

Decision 1
Decision 2
....
Decision 60

Major Depressive Disorder

Decision 1
Decision 2
….
Decision 60

Group-level distribution parameters subject to *informative* priors

Person-specific parameters drawn from group-level distributions

LBA Distribution, parameters particular to person.

Do group-level cognitive processes differ between depressed and control populations? Which processes, and by how much do they differ?

Quantitative measurement of individual participants' cognitive processes. Which of these are associated with psychological symptoms, or with lifestyle impacts?

How do the cognitive processes associate with neurophysiological measurements from these people?

Eye movement planning and "inhibition of return"
(Farrell & Ludwig, 2009)

Executive control, and attentional filtering
(Parris et al. 2012)

Effects of pre-cue and biasing information
(Serences et al. 2013)

Neurobiological network effects in ageing
(Forstmann et al., 2011)

Neurobiological accounts of decision urgency
(Forstmann et al., 2008, 2010)

THE BEST OF BOTH WORLDS

Integrating Consumer
and Perceptual Choices

OPTION 1: DEVELOP A NEW EVIDENCE ACCUMULATION MODEL

Extend an accumulator model

e.g. leaky competing accumulator model: Usher, McClelland, et al.

Extend a random walk model

e.g. Decision Field Theory: Busemeyer, Townsend, Diederich et al.

OPTION 2: EXPLOIT THE R.U.M. LINK WITH HORSE RACE MODELS

Vandekerckhove, Tuerlinckx, & Lee (2011).

van der Maas, Molenaar, Maris, Kievit & Borsboom (2011).

Tuerlinckx & De Boeck (2005).

OPTION 3: A WHOLE NEW APPROACH

Diederich’s 2N-ary choice tree model

LIMITATIONS

OPTION 1: Intractable, statistical inference very difficult, individual-person analysis difficult, require many choices.

OPTION 2: Incomplete account of response times, under-specified link with neurobiology, some limited to binary choices.

OPTION 3: Could be great!

OPTION 4: USE THE LBA

Hawkins, Marley, Heathcote, Flynn, Louviere, & Brown (in press). Integrating cognitive process and descriptive models of attitudes and preferences. *Cognitive Science*

Why? Part 1.

- Evidence accumulation models have the advantages above:
  - Neurobiological underpinnings.
  - Process-level interpretation.
  - Accounts for response times.
- But, there are many evidence accumulation models.

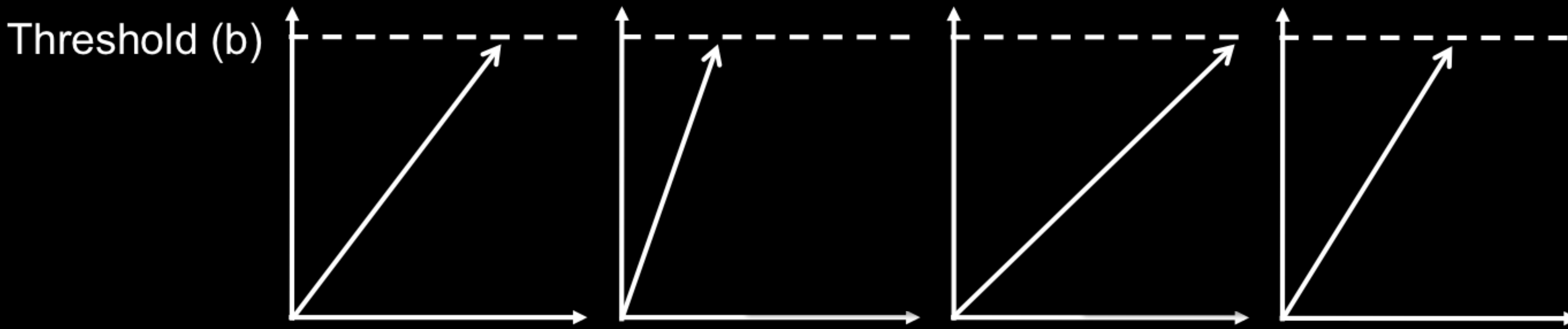
Why? Part 2.

- Why LBA?
  - Tractable.
  - Flexible. Combine evidence accumulators in complex ways, to model real choices, and tricky decision rules.
  - Powerful and practical estimation methods, including maximum likelihood and hierarchical Bayesian methods.

Challenges

Perceptual Choice	**Consumer Choice**
Mean RT: 600-900msec	Mean RT: Seconds or Minutes
Hundreds or thousands of decisions per person.	Dozens of decisions per person. At the most.
Accurate time measurement.	?
Fully factorial, small design.	Sub-factorial designs.

LBA for Consumer Choice

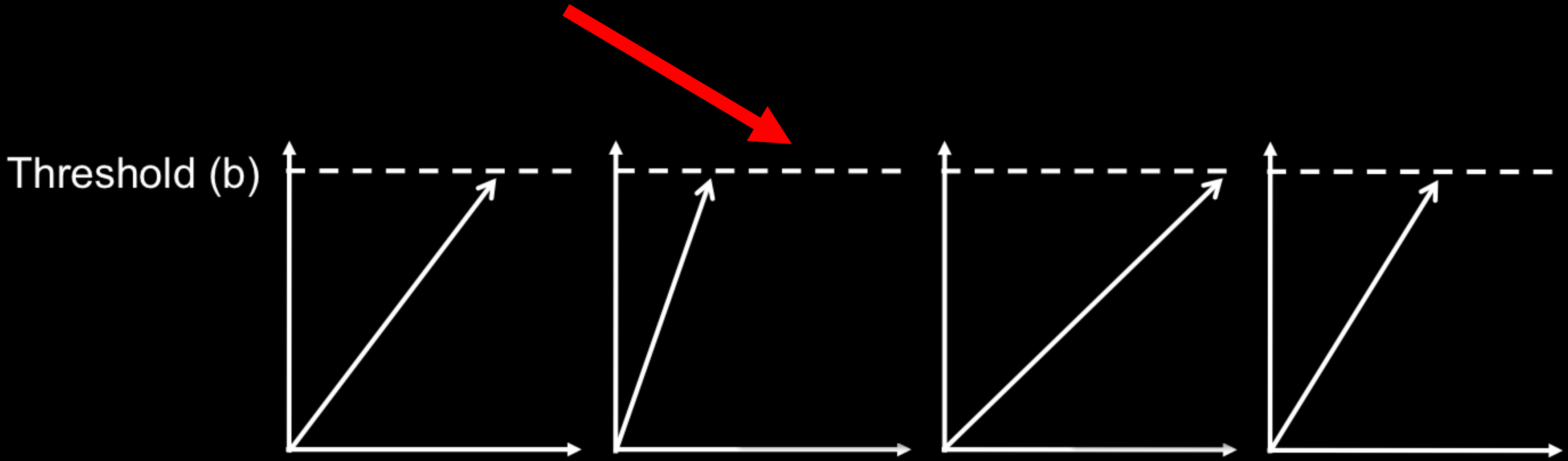


Phone1 Phone2 Phone3 Phone 4

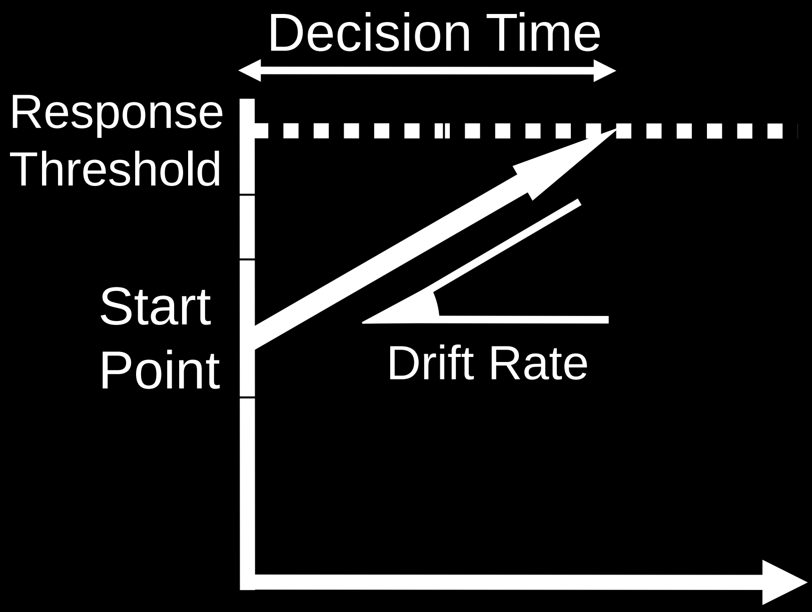
LBA for Consumer Choice

One accumulator per choice option

Fastest Finishing Accumulator = Chosen Option



Phone1 Phone2 Phone3 Phone 4



Drift Rate = Utility

Start point = Bias

Threshold = Urgency

Also: non-decision timing, two variance parameters.

BMC Medical Research Methodology

BioMed Central

Research article

Open Access

Estimating preferences for a dermatology consultation using Best-Worst Scaling: Comparison of various methods of analysis

Terry N Flynn[1], Jordan J Louviere[2], Tim J Peters[3] and Joanna Coast*[4]

EPIDEMIOLOGY AND HEALTH SERVICES RESEARCH DOI 10.1111/j.1365-2133.2006.07328.x

Preferences for aspects of a dermatology consultation

J. Coast, C. Salisbury,* D. de Berker,† A. Noble,* S. Horrocks,‡ T.J. Peters* and T.N. Flynn§

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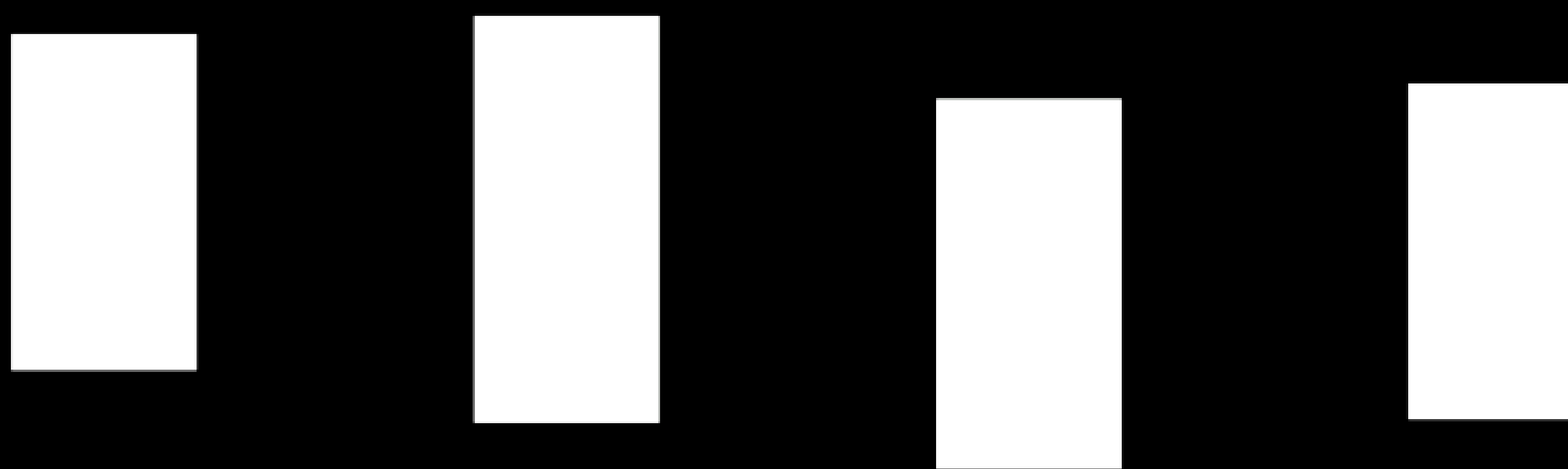
†Bristol Dermatology Centre, Bristol Royal Infirmary, Bristol, U.K.

‡Faculty of Health and Social Care, University of the West of England, Bristol, U.K.

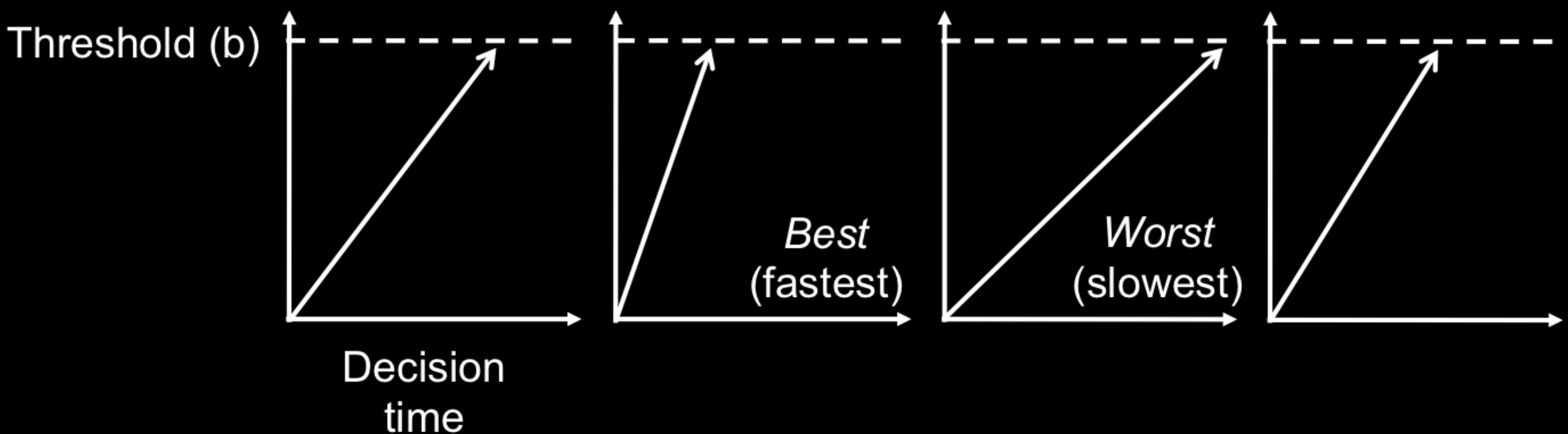
Best thing	**The appointment with the specialist**	*Worst thing*
	You will have to wait *one* month for your appointment	
	Getting to your appointment will be difficult and time-consuming	
	The consultation will be as thorough as you would like	
	The specialist is in a team led by an expert who has been treating skin complaints full-time for at least 5 years	

○ Largest 1 ○ Largest 2 ○ Largest 3 ○ Largest 4



○ Smallest 1 ○ Smallest 2 ○ Smallest 3 ○ Smallest 4

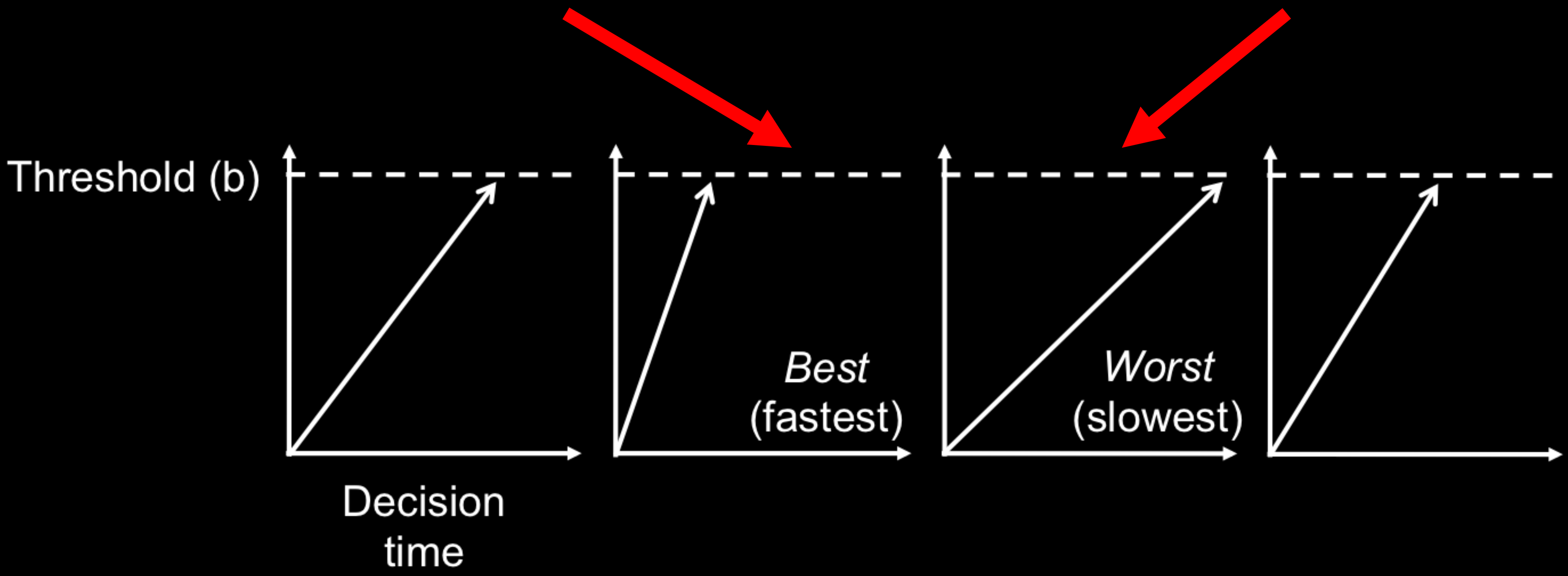
LBA Variant 1: Ranking Model



LBA Variant 1: Ranking Model

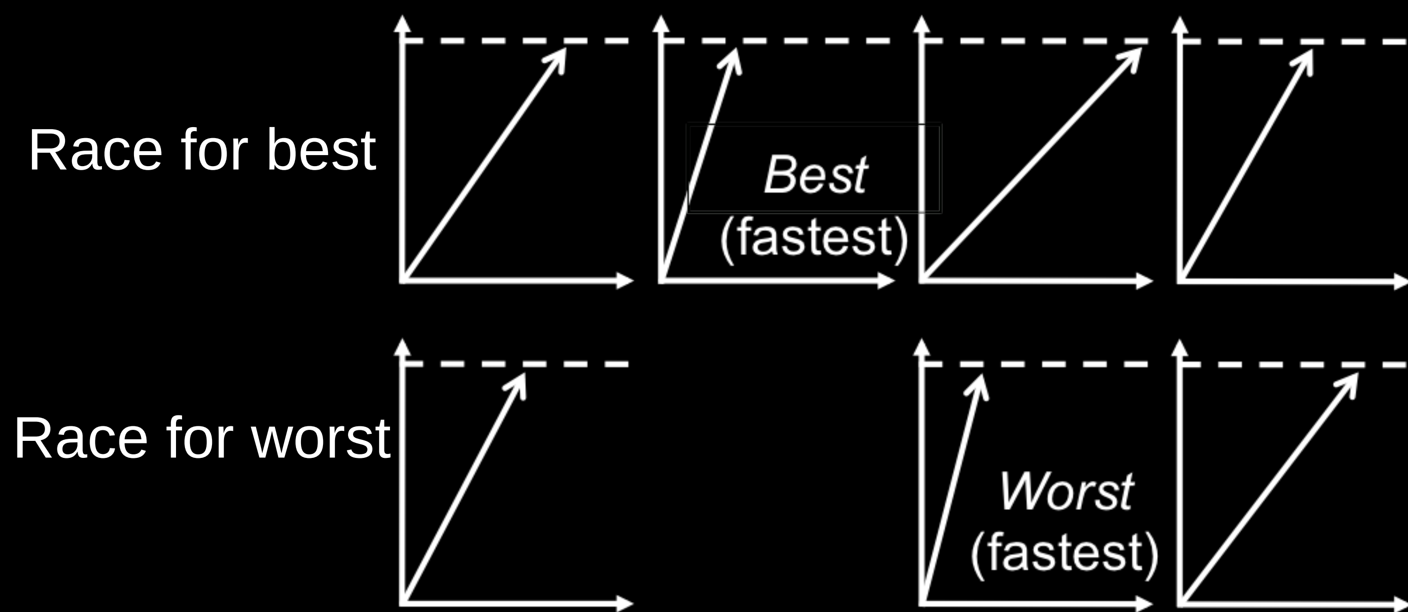
One accumulator per choice option

Fastest = Best Option, Slowest = Worst Option



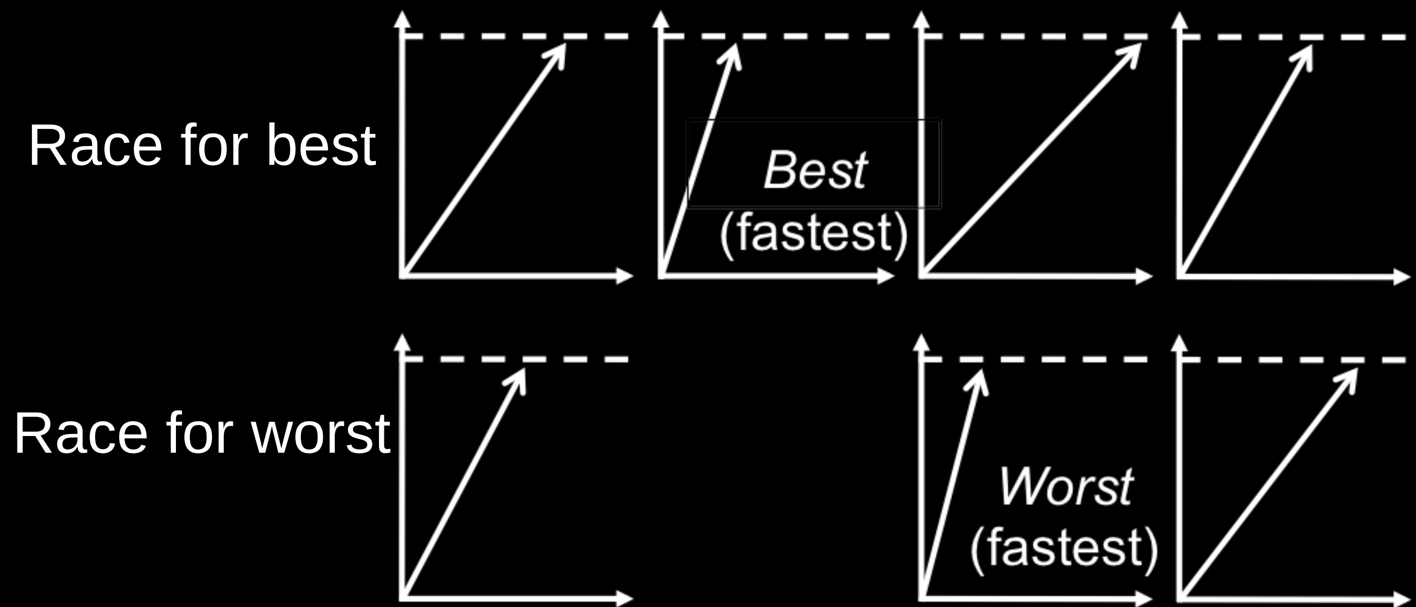
LBA Variant 2: Sequential Model

Two separate races: for “best”, for “worst”:



LBA Variant 2: Sequential Model

Two separate races: for “best”, for “worst”:
Best race according to utility.
Worst race according to 1/utility.



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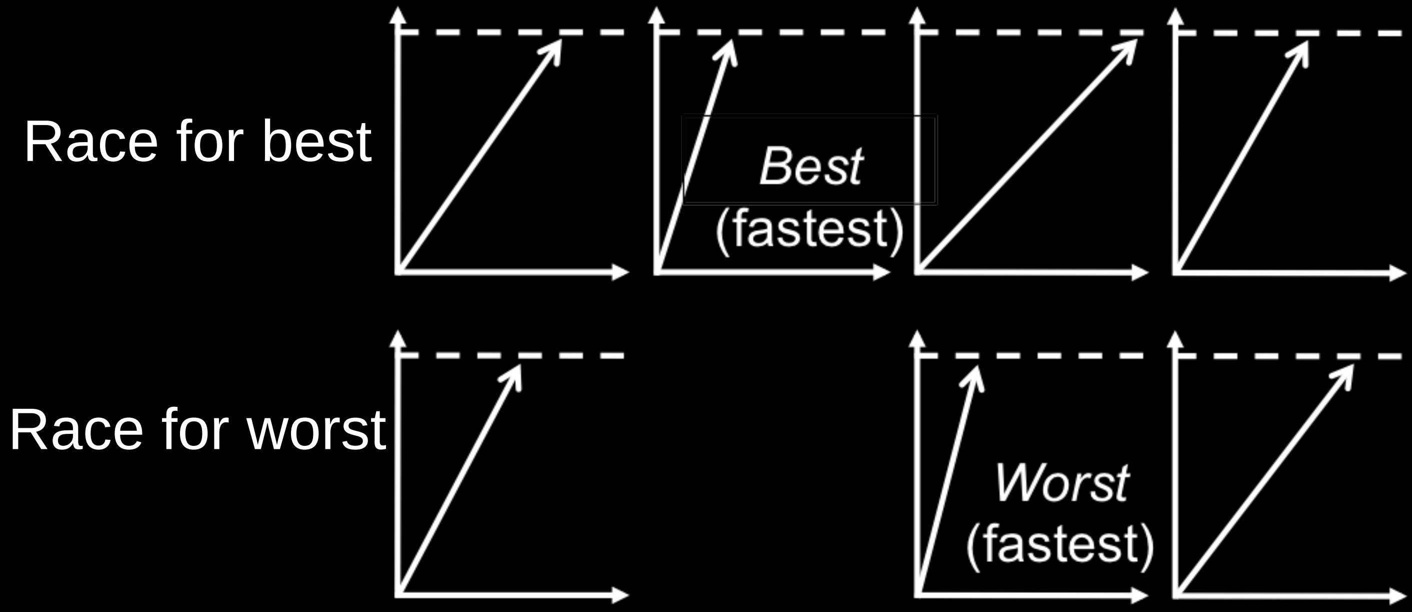
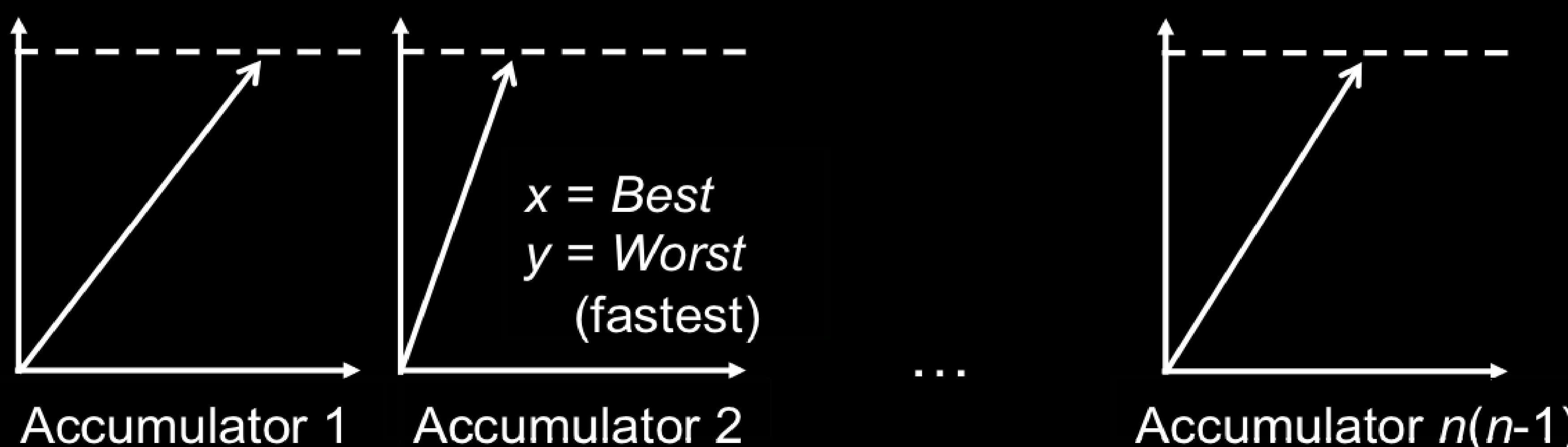


What if the same accumulator wins both races?

LBA Variant 3: Enumerated Model

For N choice options, use N*(N-1) accumulators.
Each corresponds to a best-worst **pair.**



LBA Variant 3: Enumerated Model

For N choice options, use N*(N-1) accumulators. Each corresponds to a best-worst **pair.**

Speed of accumulator *{best=i,worst=j}* is given by the ratio of utilities: $\mathbf{v}_i$ / $\mathbf{v}_j$

Easy Maths

For a single accumulator *i* finishing times have density f_i and cumulative distribution F_i.

The CDF for first passage times on a single accumulator is:

$$F_i(t) = 1 + \frac{b - A - tv_i}{A}\Phi\left(\frac{b - A - tv_i}{ts}\right) - \frac{b - tv_i}{A}\Phi\left(\frac{b - tv_i}{ts}\right) + \frac{ts}{A}\phi\left(\frac{b - A - tv_i}{ts}\right) - \frac{ts}{A}\phi\left(\frac{b - tv_i}{ts}\right)$$

The associated PDF is:

$$f_i(t) = \frac{1}{A}\left[-v_i\Phi\left(\frac{b - A - tv_i}{ts}\right) + s\phi\left(\frac{b - A - tv_i}{ts}\right) + v_i\Phi\left(\frac{b - tv_i}{ts}\right) - s\phi\left(\frac{b - tv_i}{ts}\right)\right]$$

Easy Maths

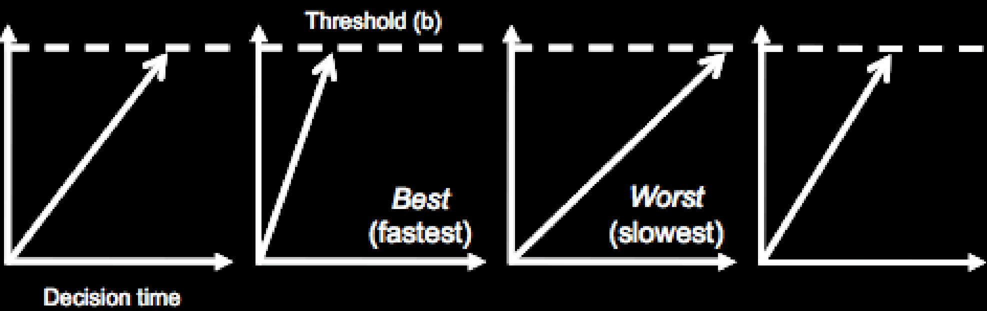
For a single accumulator i finishing times have density f_i and cumulative distribution F_i.

All the combinations are easy too, because the accumulators are independent.

Likelihood of accumulators i and j finishing at times s and t is $f_i(s)f_j(t)$

Probability of accumulator k finishing after time t is $1\text{-}F_k(t)$

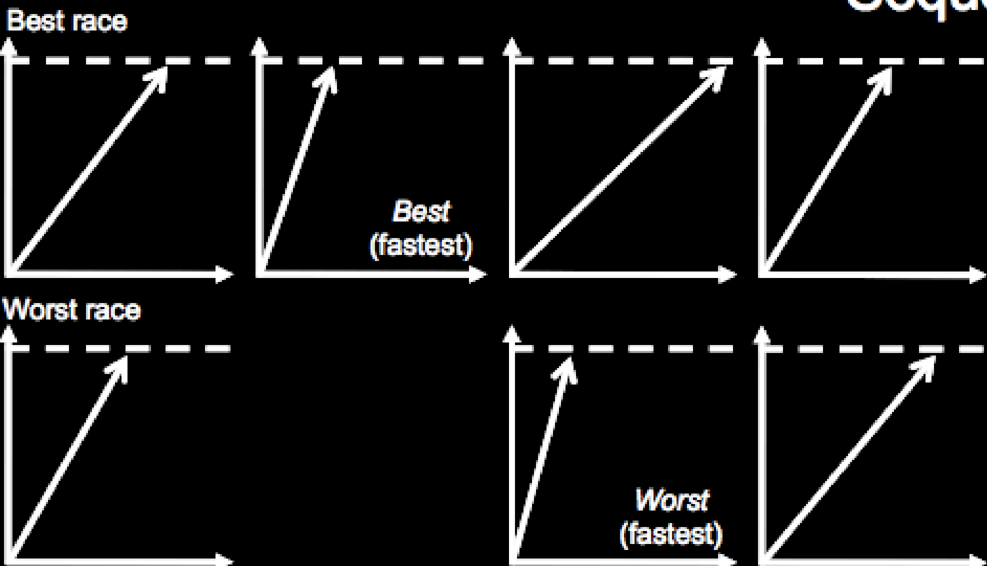
Ranking



$$bw_X(x,t;y,r) = b_x(t) \cdot b_y(r) \prod_{\substack{(p,q)\in X\times X-(x,y)\\ p\neq q}} (B_p(r) - B_q(t)).$$

$$BW_X(x,y) = \int_0^\infty \int_t^\infty bw_{x,y}(x,t;y,r)\,\mathrm{d}r\,\mathrm{d}t.$$

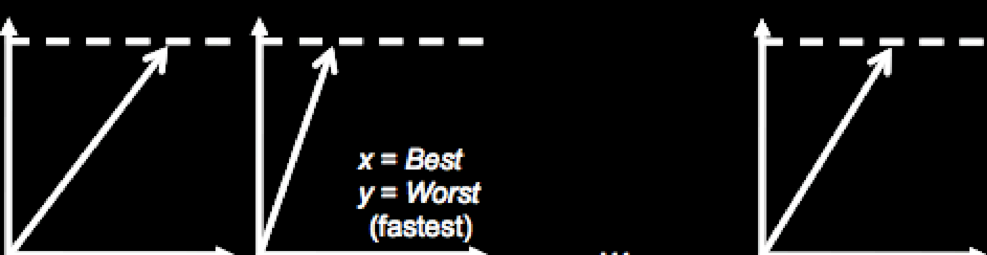
Sequential



$$B_X(x) = \int_0^\infty b_x(t) \prod_{z\in X-\{x\}} (1 - B_z(t))\,\mathrm{d}t.$$

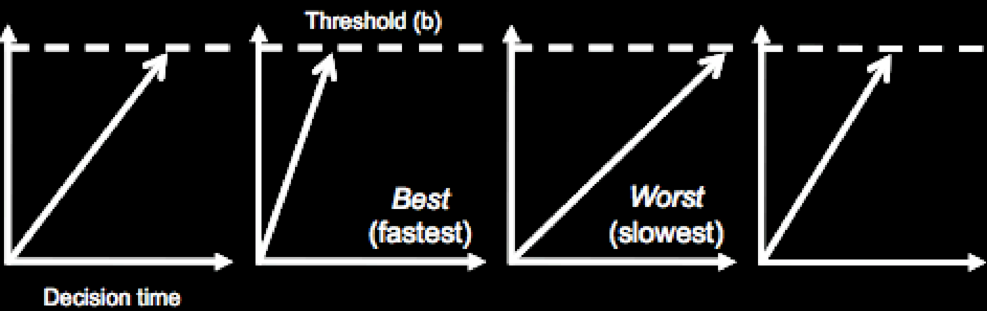
$$W_{X-\{x\}}(y) = \int_0^\infty w_y(r) \prod_{z\in X-\{x,y\}} (1 - W_z(r))\,\mathrm{d}r.$$

Enumerated



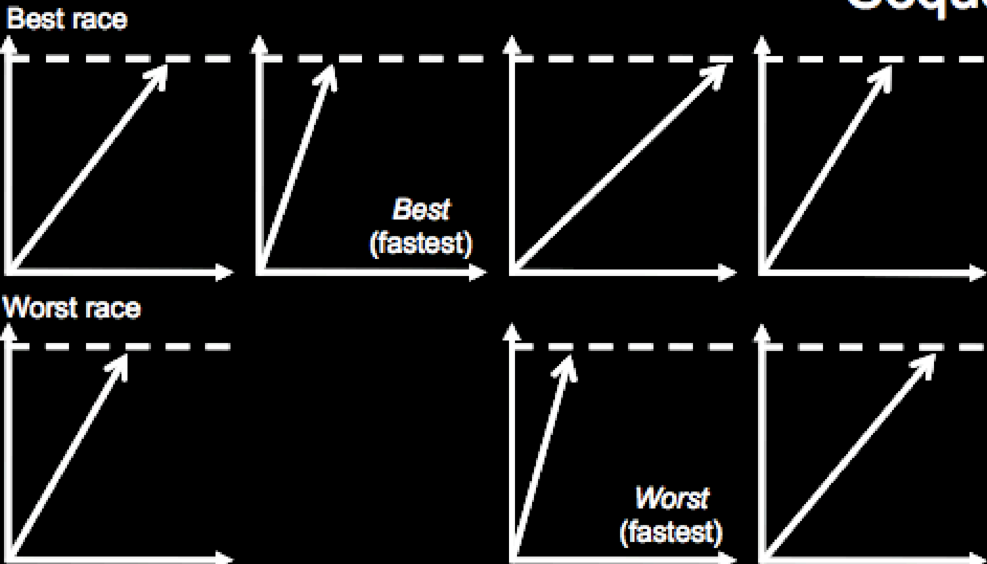
$$BW_X(x,y) = \int_0^\infty bw_{(x,y)}(t) \prod_{\substack{(p,q)\in X\times X-(x,y)\\ p\neq q}} (1 - BW_{(p,q)}(t))\,\mathrm{d}t.$$

Ranking



$$bw_X(x,t;y,r) = b_x(t)\cdot b_y(r) \prod_{\substack{(p,q)\in X\times X-(x,y)\\ p\neq q}} (B_p(r)-B_q(t)).$$

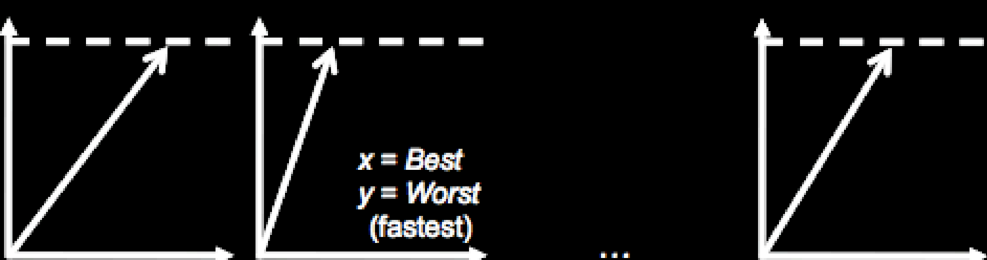
Sequential



$$b_x(t) \prod_{z\in X-\{x\}} (1-B_z(t))$$

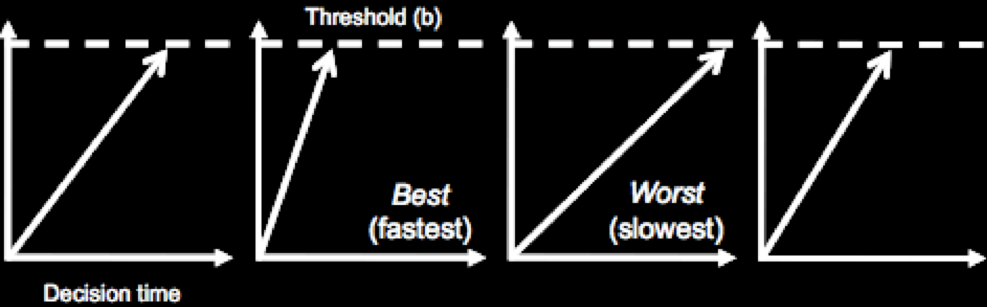
$$w_y(r) \prod_{z\in X-\{x,y\}} (1-W_z(r))$$

Enumerated



$$bw_{(x,y)}(t) \prod_{\substack{(p,q)\in X\times X-(x,y)\\ p\neq q}} (1-BW_{(p,q)}(t))$$

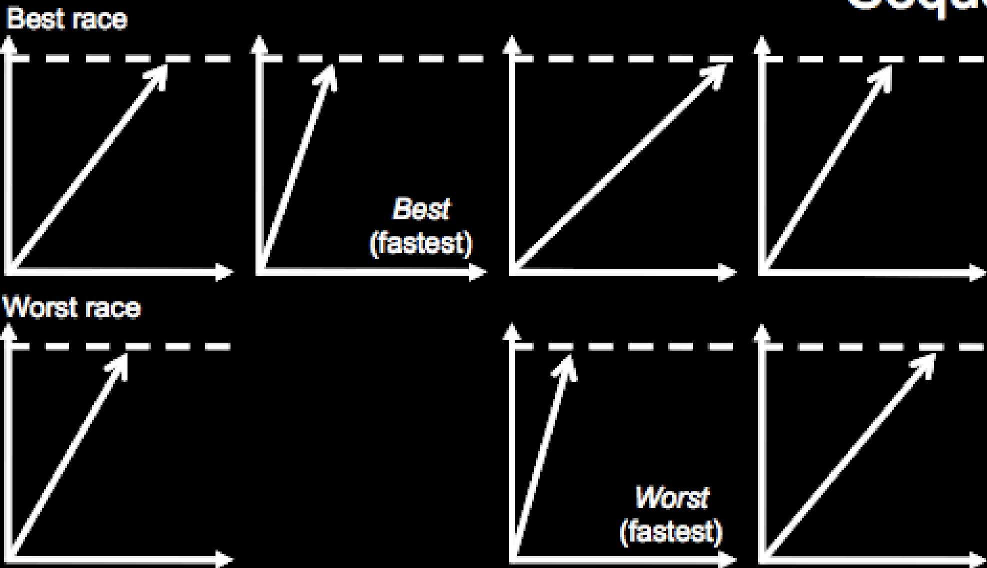
Ranking



$$bw_X(x,t;y,r) = b_x(t)\cdot b_y(r) \prod_{\substack{(p,q)\in X\times X-(x,y)\\ p\neq q}} (B_p(r) - B_q(t)).$$

$$BW_X(x,y) = \int_0^\infty \int_t^\infty bw_{x,y}(x,t;y,r)\,\mathrm{d}r\,\mathrm{d}t.$$

Sequential



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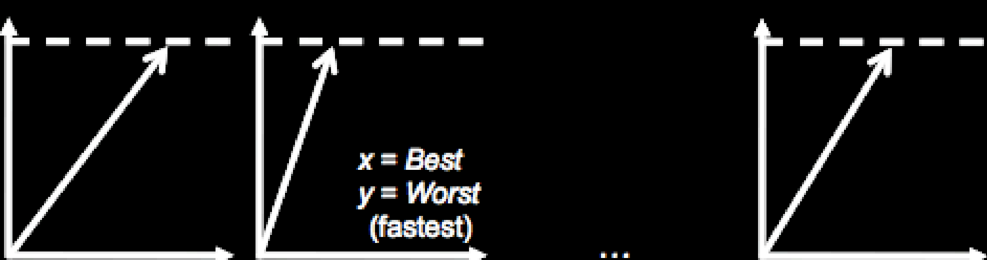
Enumerated



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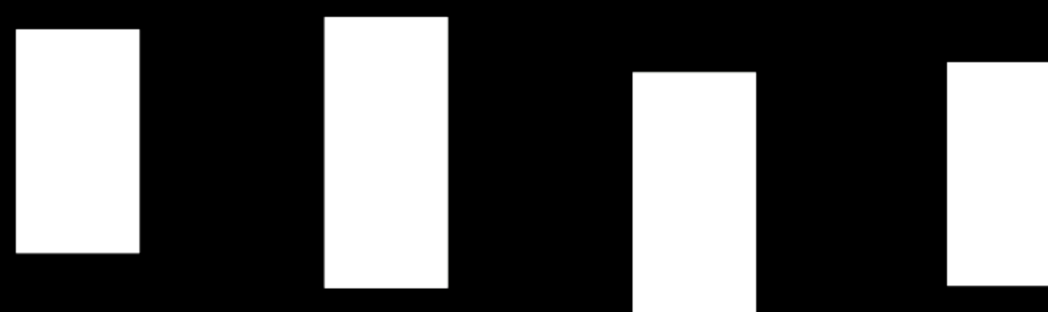
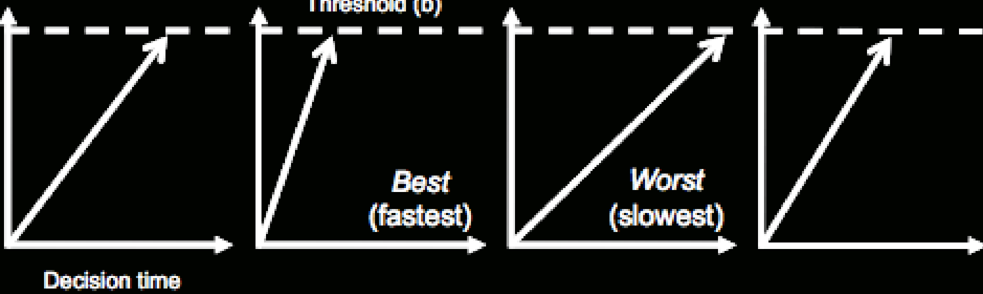
○ Largest 1 ○ Largest 2 ○ Largest 3 ○ Largest 4



○ Smallest 1 ○ Smallest 2 ○ Smallest 3 ○ Smallest 4

Ranking



$$bw_X(x,t;y,r) = b_x(t) \cdot b_y(r) \prod_{\substack{(p,q)\in X\times X-(x,y)\\ p\neq q}} (B_p(r) - B_q(t)).$$

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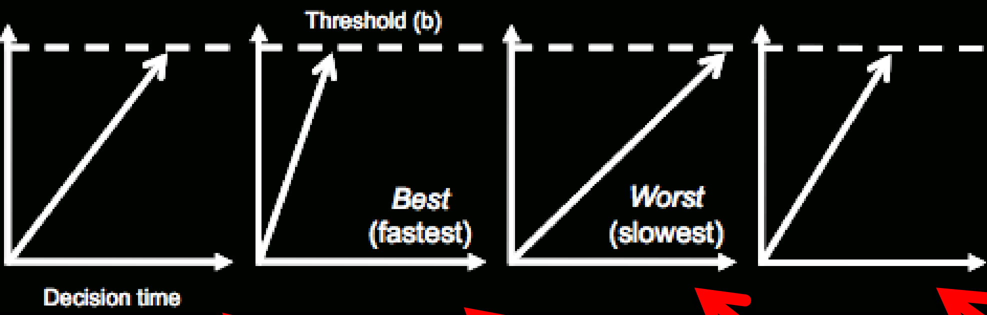
Largest 1 Largest 2 Largest 3 Largest 4

Smallest 1 Smallest 2 Smallest 3 Smallest 4

Ranking



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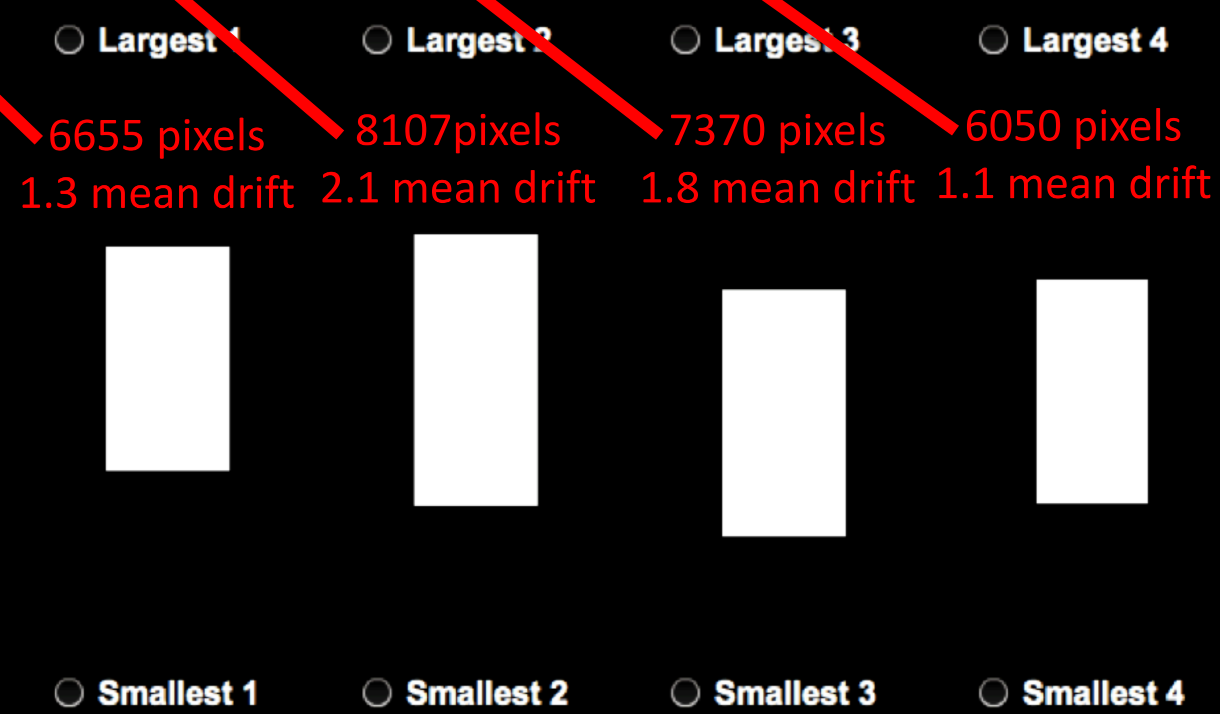
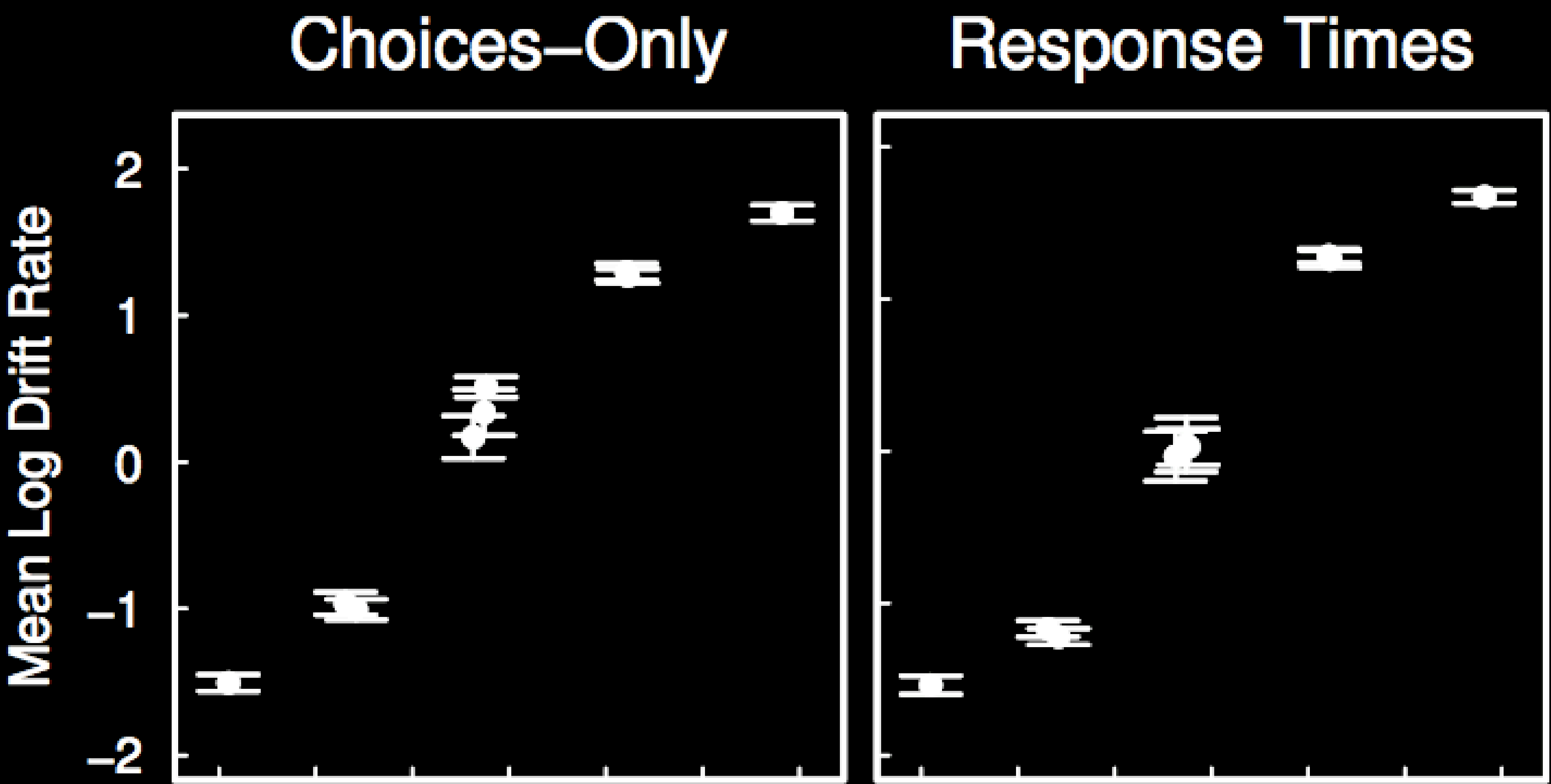
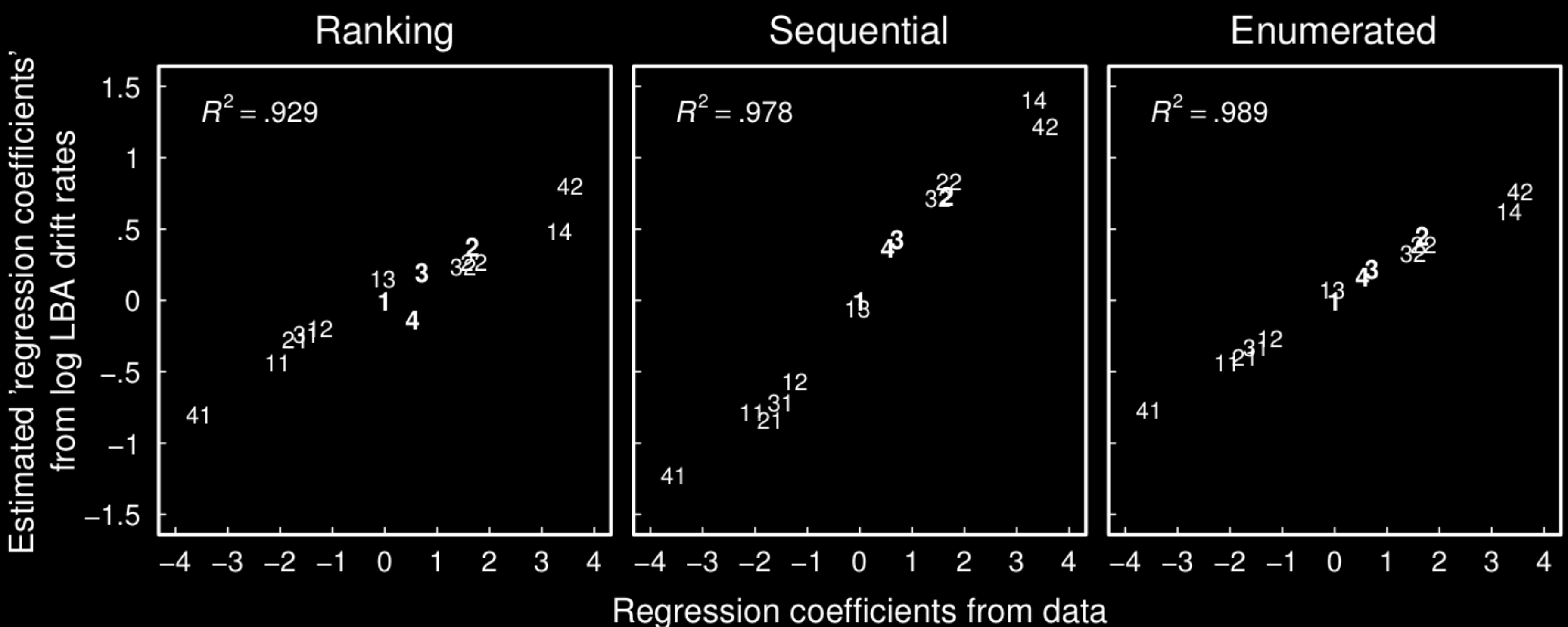




Rectangle Area in Pixels

Best thing	**The appointment with the specialist**	*Worst thing*
	You will have to wait *one* month for your appointment	
	Getting to your appointment will be difficult and time-consuming	
	The consultation will be as thorough as you would like	
	The specialist is in a team led by an expert who has been treating skin complaints full-time for at least 5 years	

LBA's Drift Rates are proportional to R.U.M.'s Estimates



Journal of Mathematical Psychology 56 (2012) 24–34

Contents lists available at SciVerse ScienceDirect

Journal of Mathematical Psychology

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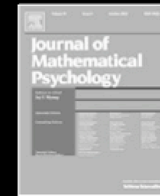
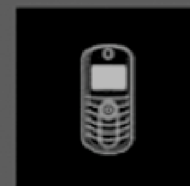
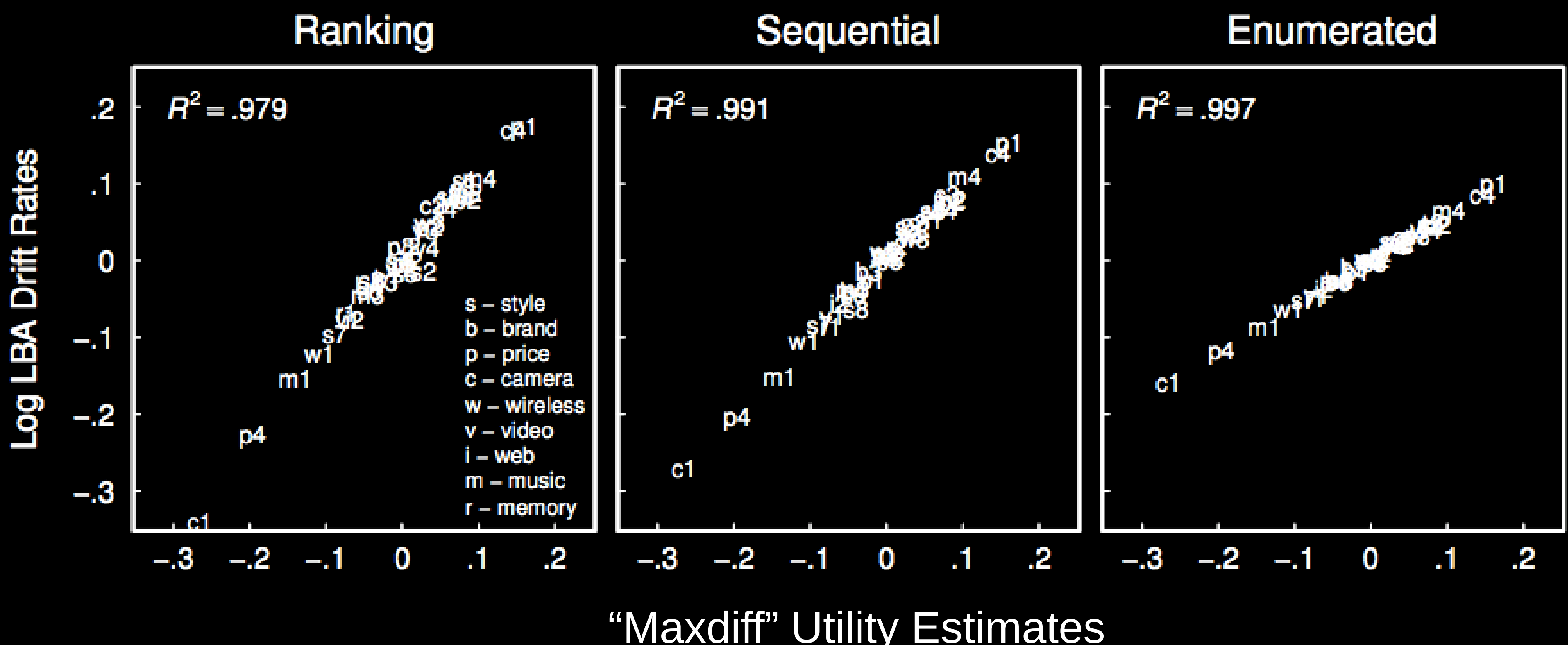


Models of best–worst choice and ranking among multiattribute options (profiles)

A.A.J. Marley [a,*], D. Pihlens [b]

	Phone 1	Phone 2	Phone 3	Phone 4
Phone Style	 Clam or flip phone	 Candy Bar or straight phone	 Swivel flip	 PDA phone with touch screen input
Handset Brand	A	B	C	D
Price	$49.00	$199.00	$249.00	$129.00
Built-in Camera	No camera	5 megapixel camera	2 megapixel camera	3 megapixel camera
Wireless Connectivity	No Bluetooth or WiFi connectivity	Bluetooth and WiFi connectivity	WiFi connectivity	Bluetooth connectivity
Video Capability	No video recording	Video recording (up to 1 hour)	Video recording (more than 1 hour)	Video recording (up to 15 minutes)
Internet Capability	Internet Access	Internet Access	No Internet access	No Internet access
Music Capability	No music capability	MP3 Music Player only	FM Radio only	MP3 Music Player and FM Radio
Handset Memory	64 MB built-in memory	2 GB built-in memory	512 MB built-in memory	4 GB built-in memory

LBA's Drift Rates are proportional to R.U.M.'s Estimates



So Why Bother?

- Replacing R.U.M. with LBA has only those advantages detailed before:
  - Neurobiological underpinnings.
  - Process-level interpretation.
  - Accounts for response times.
- Maybe also …
  - Easier communication with stakeholders.
  - Estimate variance & bias parameters.

?

RT can Answer some Cognitive Questions: Part 1

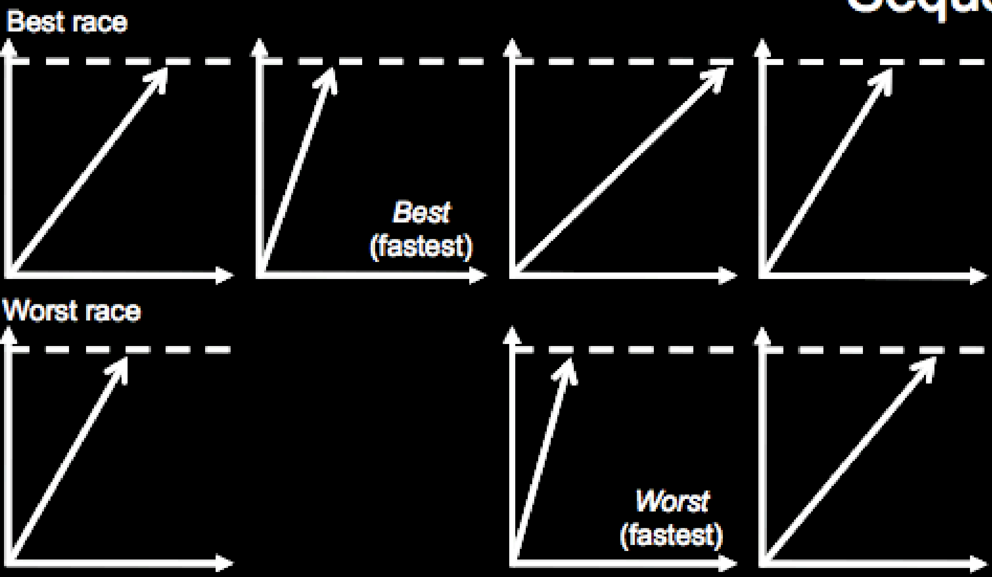
Remember:

Best race according to utility.
Worst race according to 1/utility.

Test by nested model comparison (using LR test, or BIC, or even Bayes Factors)

RT can Answer some Cognitive Questions: Part 1

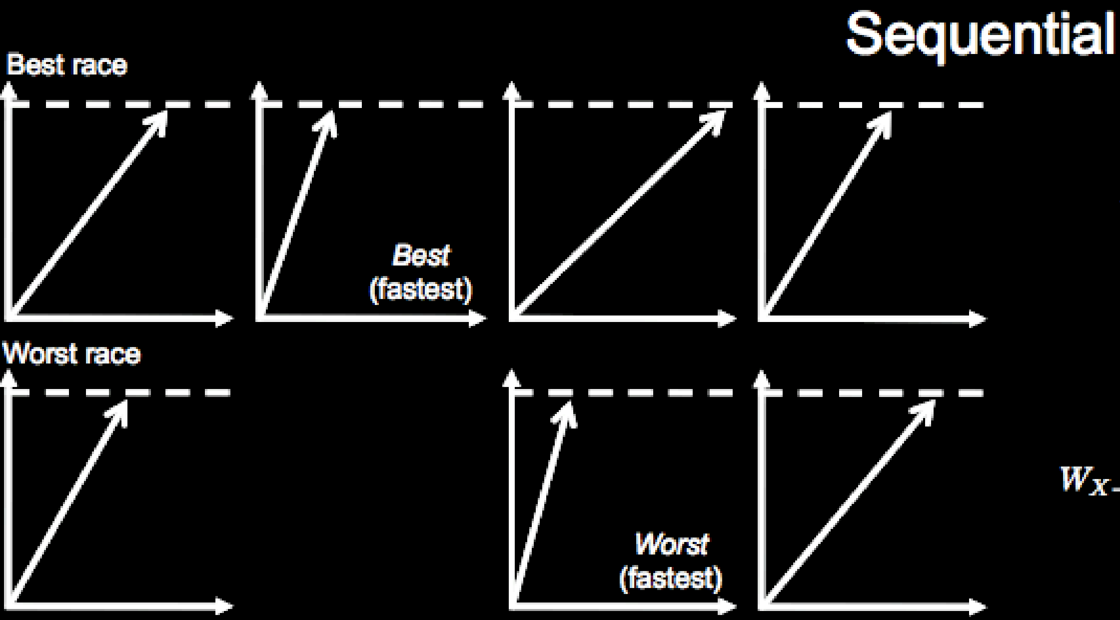
Sequential



$$B_X(x) = \int_0^\infty b_x(t) \prod_{z \in X - \{x\}} (1 - B_z(t))\,\mathrm{d}t.$$

$$W_{X-\{x\}}(y) = \int_0^\infty w_y(r) \prod_{z \in X - \{x,y\}} (1 - W_z(r))\,\mathrm{d}r.$$

RT can Answer some Cognitive Questions: Part 1



$$B_X(x) = \int_0^\infty b_x(t) \prod_{z \in X - \{x\}} (1 - B_z(t))\, \mathrm{d}t.$$

$$W_{X-\{x\}}(y) = \int_0^\infty w_y(r) \prod_{z \in X - \{x,y\}} (1 - W_z(r))\, \mathrm{d}r.$$

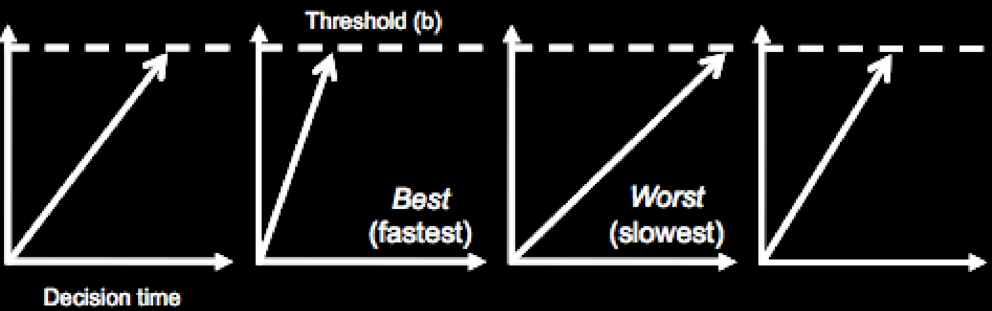
Best race according to utility.

Worst race according to 1/utility. ✔

RT can Answer some Cognitive Questions: Part 2

Best thing	**The appointment with the specialist**	***Worst thing***
	You will have to wait *one* month for your appointment	
	Getting to your appointment will be difficult and time-consuming	
	The consultation will be as thorough as you would like	
	The specialist is in a team led by an expert who has been treating skin complaints full-time for at least 5 years	

Ranking



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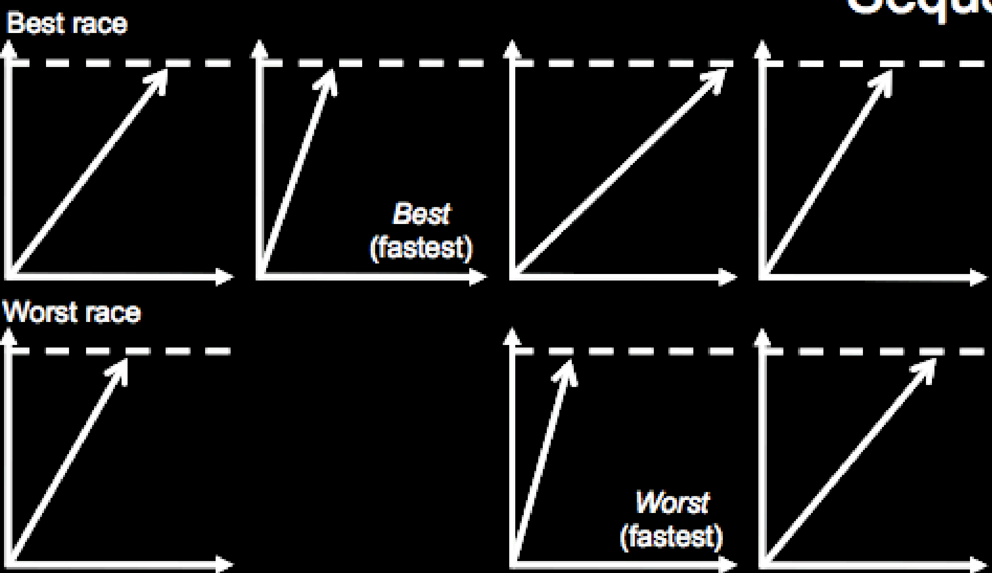
$$BW_X(x,y)=\int_0^\infty\int_t^\infty bw_{x,y}(x,t;y,r)\,\mathrm{d}r\,\mathrm{d}t.$$

Sequential



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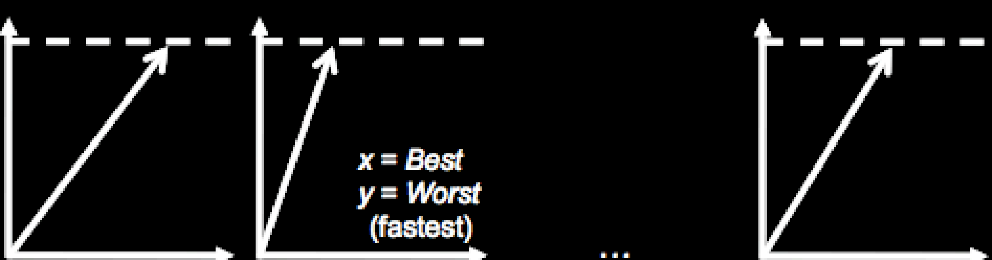
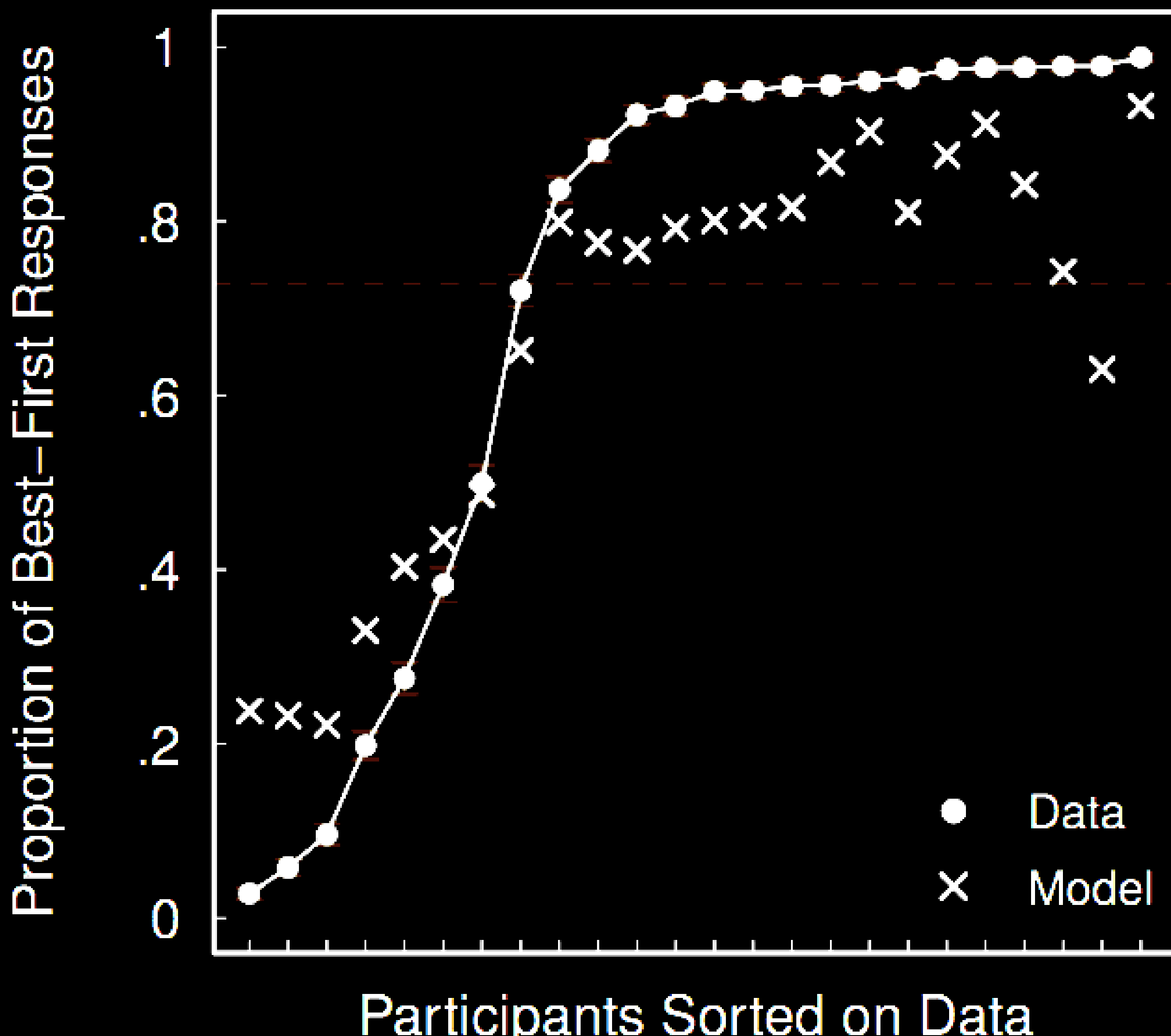
$$W_{X-\{x\}}(y)=\int_0^\infty w_y(r)\prod_{z\in X-\{x,y\}}(1-W_z(r))\,\mathrm{d}r.$$

Enumerated

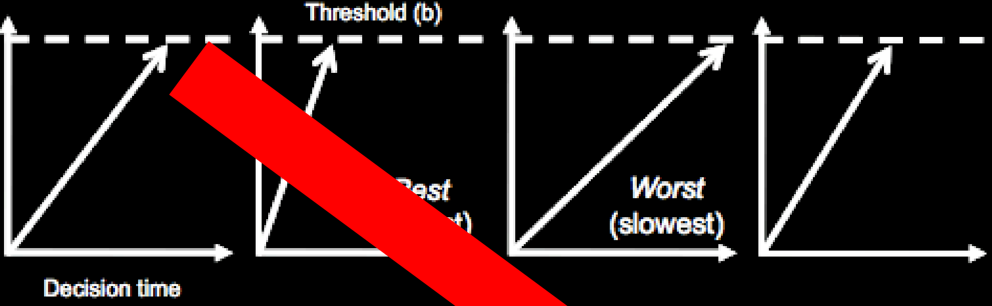


$$BW_X(x,y)=\int_0^\infty bw_{(x,y)}(t)\prod_{\substack{(p,q)\in X\times X-(x,y)\\ p\neq q}}(1-BW_{(p,q)}(t))\,\mathrm{d}t.$$

RT can Answer some Cognitive Questions: Part 2



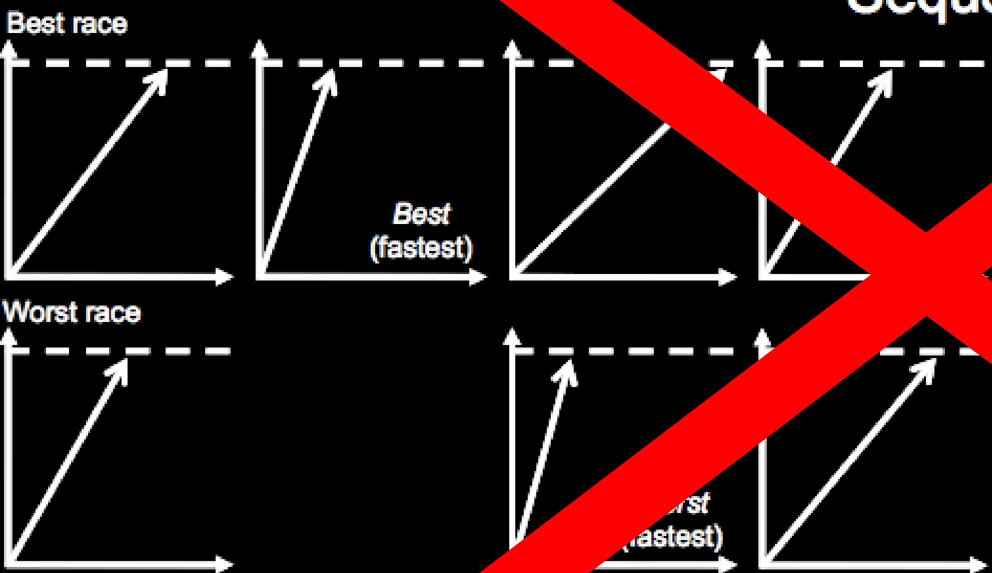
Ranking



$$bw_X(x,t;y,r) = b_x(t)\cdot b_y(r) \prod_{(p,q)\in X\times X [illegible]} [illegible]_p(r) - B_q(t)).$$

$$BW_X(x,y) = \int_0^\infty \int_t^\infty bw[illegible], y, r)\, \mathrm{d}r\, \mathrm{d}t.$$

Sequential



$$[illegible]_X(x) = \int_0^\infty b_x(t) \prod_{z\in X-\{x\}} (1 - B_z(t))\, \mathrm{d}t.$$

$$W_X[illegible]) = \int_0^\infty w_y(r) \prod_{z\in X-\{x,y\}} (1 - W_z(r))\, \mathrm{d}r.$$

Enumerated

$$BW_X(x,y) = \int_0^\infty bw_{(x,y)}(t) \prod_{\substack{(p,q)\in X\times X-(x,y)\\ p\neq q}} (1 - BW_{(p,q)}(t))\, \mathrm{d}t.$$

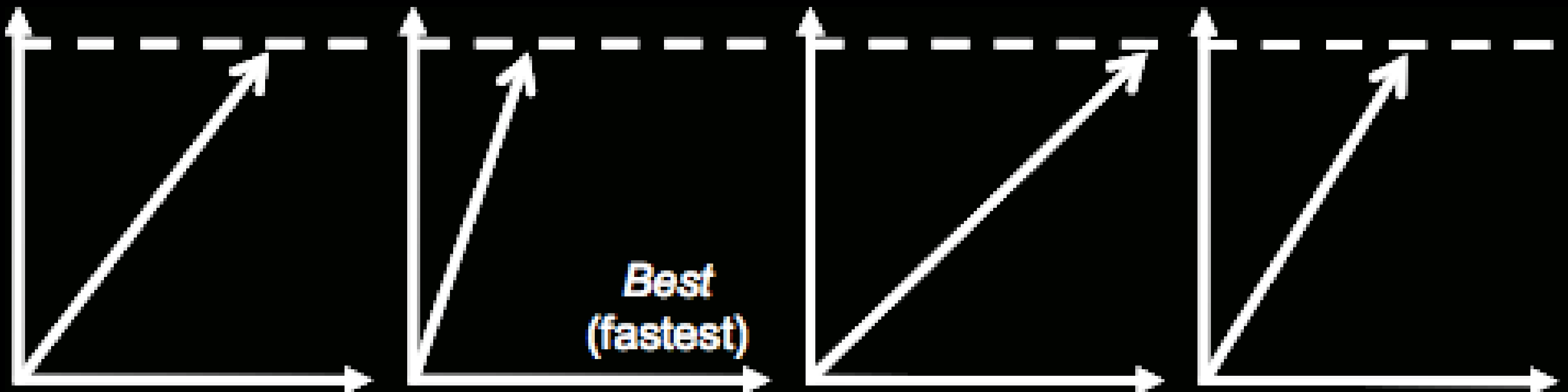
Mixture Model?

Some complicated race?

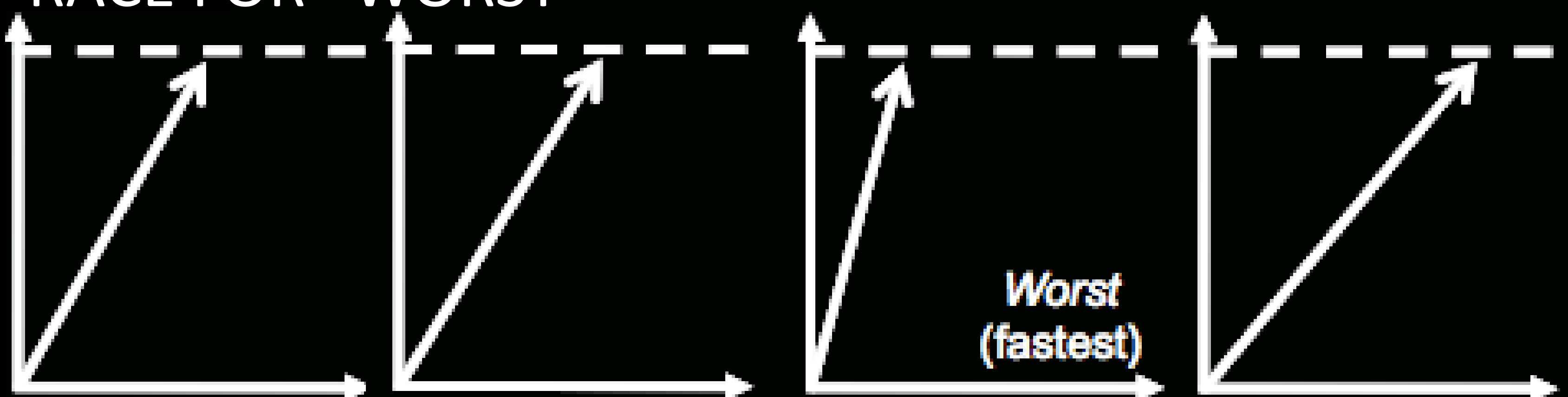
Some complicated decision rule?

A *parallel* race

RACE FOR “BEST”



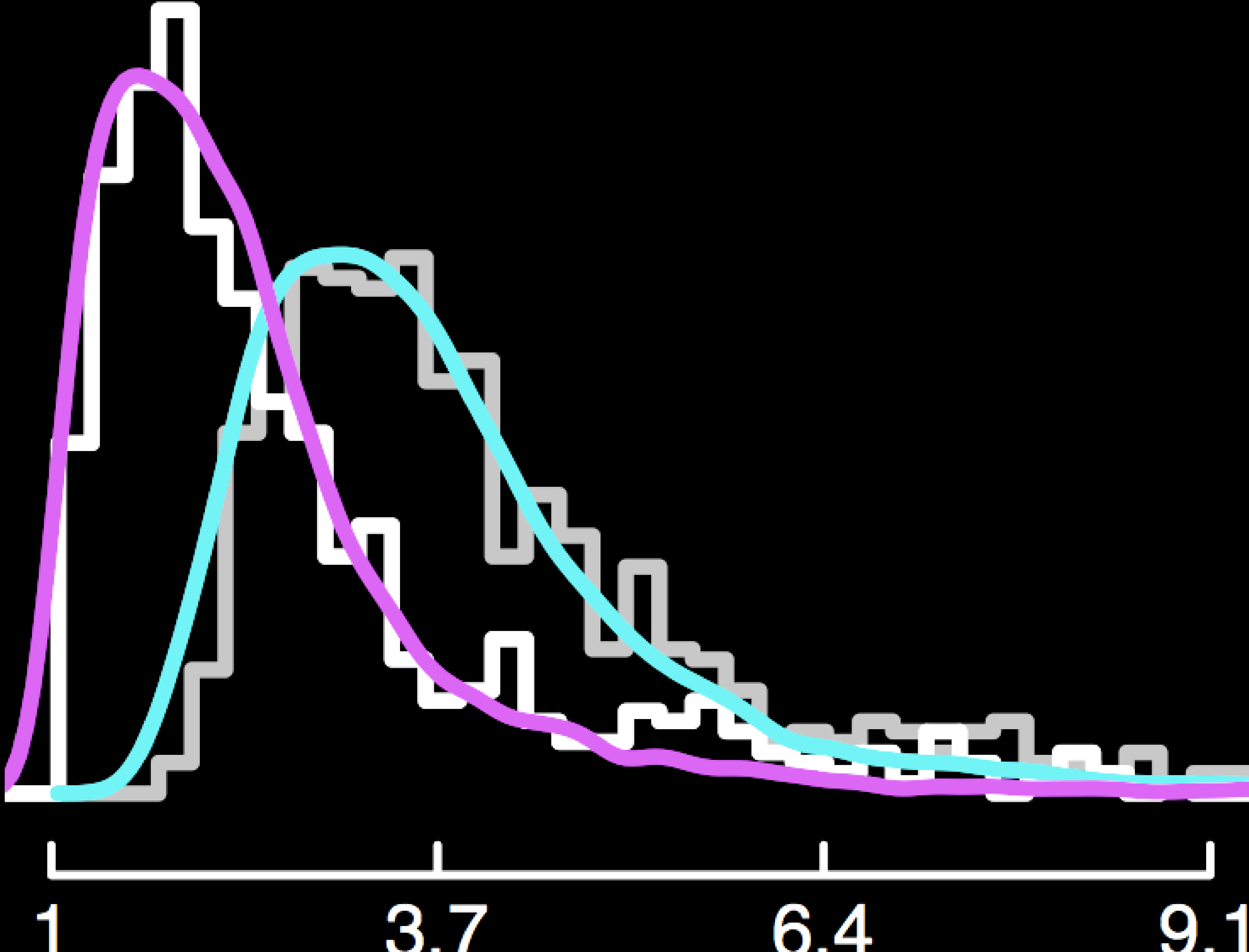
RACE FOR “WORST”

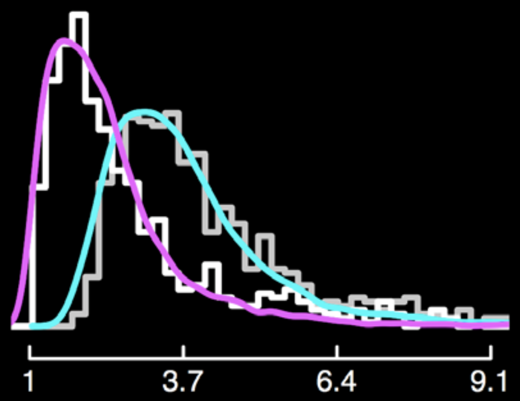


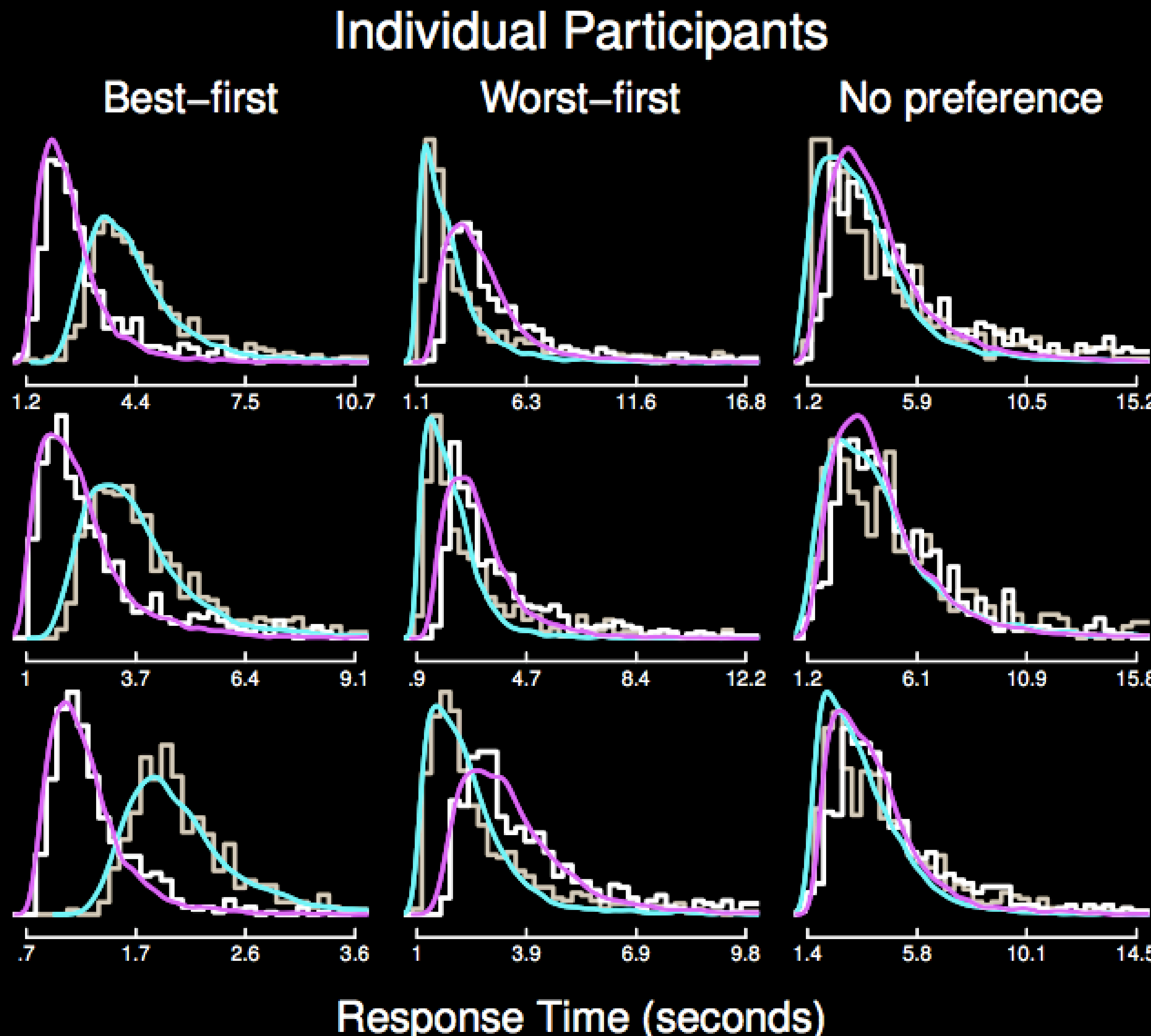
A *parallel* race

The probability of a choice of the option x as best at time t and option y as worst at time r, where no constraint exists between t and r, is given as the product of the individual likelihoods of the best and worst races,

$$bw_X(x, t; y, r) = b_x(t) \prod_{z \in X - \{x\}} (1 - B_z(t)) \;\cdot\; w_y(r) \prod_{z \in X - \{y\}} (1 - W_z(r)).$$


1
3.7
6.4
9.1


1
3.7
6.4
9.1


Individual Participants
Best–first
Worst–first
No preference
1.2
4.4
7.5
10.7
1.1
6.3
11.6
16.8
1.2
5.9
10.5
15.2
1
3.7
6.4
9.1
.9
4.7
8.4
12.2
1.2
6.1
10.9
15.8
.7
1.7
2.6
3.6
1
3.9
6.9
9.8
1.4
5.8
10.1
14.5
Response Time (seconds)

Conclusions

- Random utility models have some limitations.
  - But these are often not problematic.

- The link between the horse-race and R.U.M. can be updated, using a modern evidence accumulation model – the linear ballistic accumulator.

Conclusions

- Swapping RUM for LBA:
  - Keeps statistical tractability.
  - Does not change existing conclusions.
  - Provides a cognitive process account.
  - Brings neurophysiological detail & structure.
  - Accounts for response time data.

Conclusions

- Response time data:
  - Should *never* be considered in isolation!
  - Can answer cognitive questions.
  - Allow bias and variance parameters to be measured.

Conclusions

- In our consumer-choice applications so far:
  - Best-worst scaling and best-worst selection are consistent with best-only.
  - The most plausible cognitive account assumes parallel races for best and for worst choices.
  - The same modelling framework has worked for perceptual choices (rectangles), consumer choices (phones) and health choices (dermatologists).

end