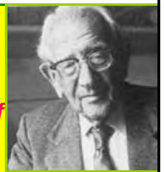
August 13th 2008

What's New, What's Possible, What's Coming ...

Jonathan Borwein, FRSC www.cs.dal.ca/~jborwein
Canada Research Chair in Collaborative Technology, Dalhousie
Visiting Professor Laureate, University of Newcastle

"intuition comes to us much earlier and with much less outside influence than formal arguments which we cannot really understand unless we have reached a relatively high level of logical experience and sophistication."



George Polya
1887-1985

Revised 13/08/2008

Outline of Presentation

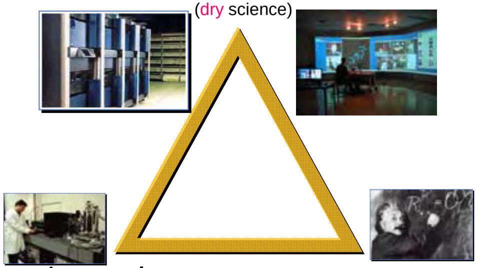
I. The Changing Research Landscape

II. New Ways of Doing Mathematics

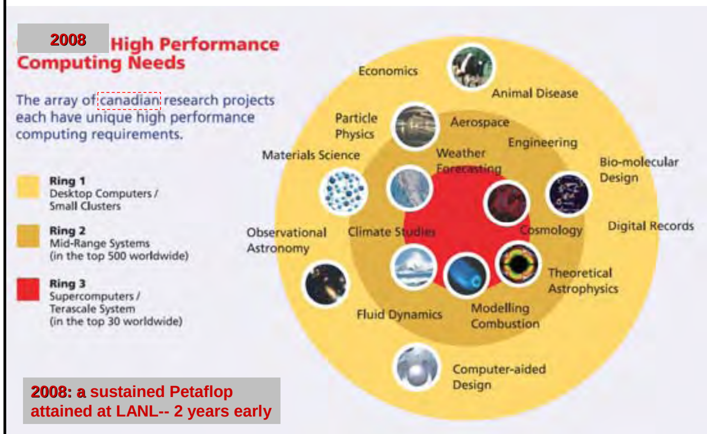
III. New Ways of Seeing Mathematics

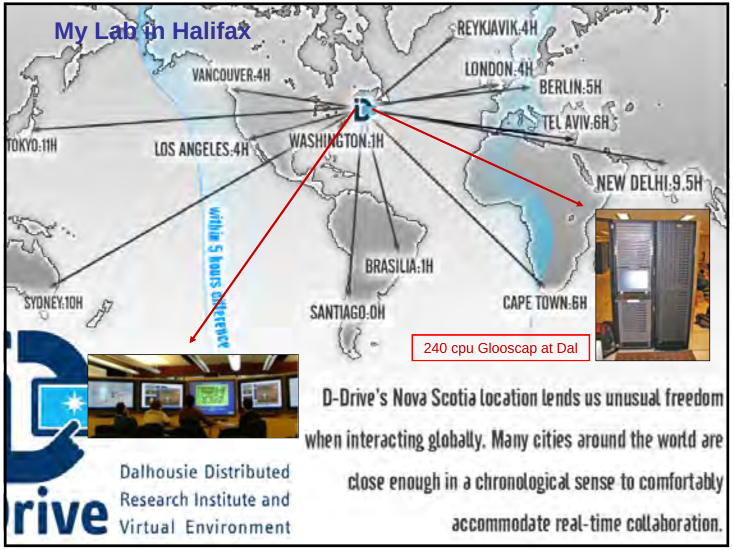
IV. Amazing New Web Services

I. Changing Research Landscape: a new triangle



HPC Needs in Canada or Oz





Moore's Law and its Implications

"The complexity for minimum component costs has increased at a rate of roughly a factor of two per year ...

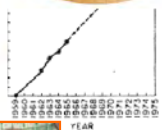
- now taken as "every 18 months to 2 years"

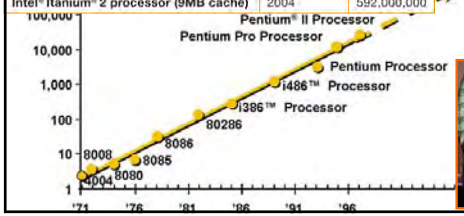
Certainly over the short term this rate can be expected to continue, if not to increase. Over the longer term, the rate of increase is a bit more uncertain, although there is no reason to believe it will not remain nearly constant for at least 10 years. That means by 1975, the number of components per integrated circuit for minimum cost will be 65,000. I believe that such a large circuit can be built on a single wafer.

Gordon Moore (**Intel**) "Cramming more components onto Electronic Circuits", *Electronics Magazine* 19 April 1965

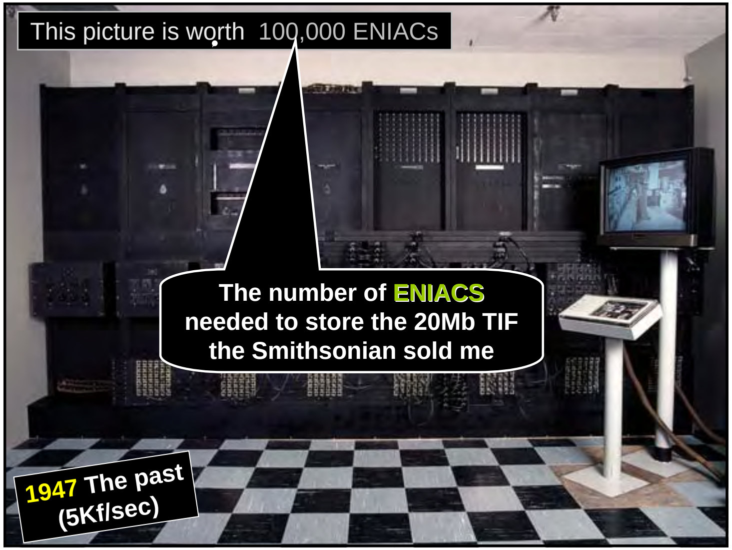
Unprecedented and expected to continue for 10-20 years.

Microprocessor	Year of Introduction	Transistors
4004	1971	2,300
8008	1972	2,500
8080	1974	4,500
8086	1978	29,000
Intel286	1982	134,000
Intel386™ processor	1985	275,000
Intel486™ processor	1989	1,200,000
Intel® Pentium® processor	1993	3,100,000
Intel® Pentium® II processor	1997	7,500,000
Intel® Pentium® III processor	1999	9,500,000
Intel® Pentium® 4 processor	2000	42,000,000
Intel® Itanium® processor	2001	25,000,000
Intel® Itanium® 2 processor	2003	220,000,000
Intel® Itanium® 2 processor (9MB cache)	2004	592,000,000



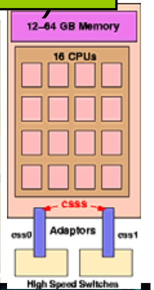


Moore's Law
1965 to 2005



NERSC's 6000 cpu Seaborg in 2004 (10Tflops/sec)

- we need new software paradigms for `bigga-scale' hardware



The near-present

Mathematical Immersive Reality in Vancouver

IBM BlueGene/L at LANL

System (64 cabinets, 64x32x32)

IBM Computer Achieves Petaflop Performance

6/9/2008

A National Nuclear Security Administration (NNSA) supercomputer has achieved an operational rate of 1,000 trillion calculations per second, or 1 petaflop, making the Roadrunner -- which the NNSA commissioned IBM Corp. to build in 2006 for around $130 million -- the world's fastest computer, the agency announced today.

2.8/5.6 GF/s 4 MB

0.5 GB DDR

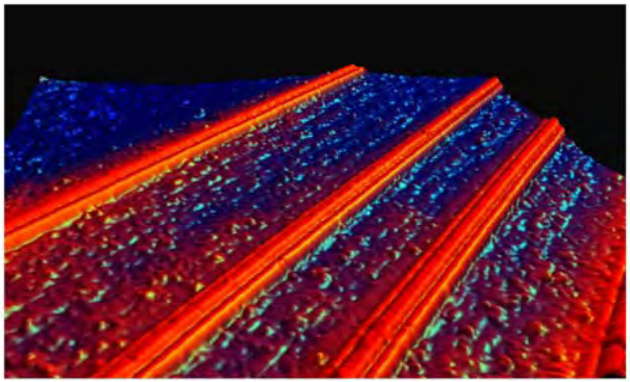
2^{17} cpu's: Oct 2007 ran Linpack benchmark at over **596 Tflop /sec**

(5 x Canada or 8 x Oz)

The future 2005-2010

Things we can't model here include:

Self assembling wires 2nm apart (HP Labs)



 Simulation by Keuckes-Williams

"It says it's sick of doing things like inventories and payrolls, and it wants to make some break-throughs in astrophysics."

II. New Ways of Doing Math

- and **related subjects**: Computer Science, Statistics, Engineering, all Sciences, *every* other subject …. for learning or for research
  - Experimentally on the Computer
  - Visual or Haptic or Acoustic Output
  - Simulations and Emersions
  - With Web-services, Databases, Wikis, …
    - Marvelous support tools for the classroom
- also **New Ways** of **Collaborating**

Jon Borwein's Math Resource Portal

The following is a list of useful math tools.

Utilities

1. ISC2.0: The Inverse Symbolic Calculator
2. EZ Face : An interface for evaluation of Euler sums and Multiple Zeta Values
3. 3D Function Grapher
4. GraPHedron: Automated and computer assisted conjectures in graph theory
5. Julia and Mandelbrot Set Explorer
6. Embree-Trefethen-Wright pseudospectra and eigenproblem

Reference

7. The On-Line Encyclopedia of Integer Sequences
8. Finch's Mathematical Constants
9. The Digital Library of Mathematical Functions
10. The Prime Pages

Content

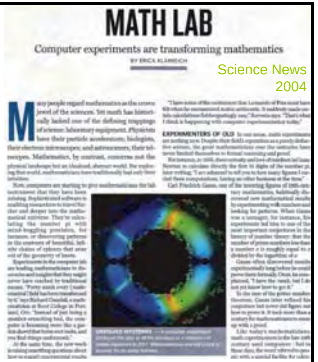
11. Experimental Mathematics Website
12. Wolfram Mathworld
13. Planet Math
14. Numbers, Constants, and Computation
15. Wikipedia: Mathematics

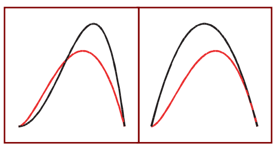
ICCOPT 2007 Short Course

16. Jon's Lectures

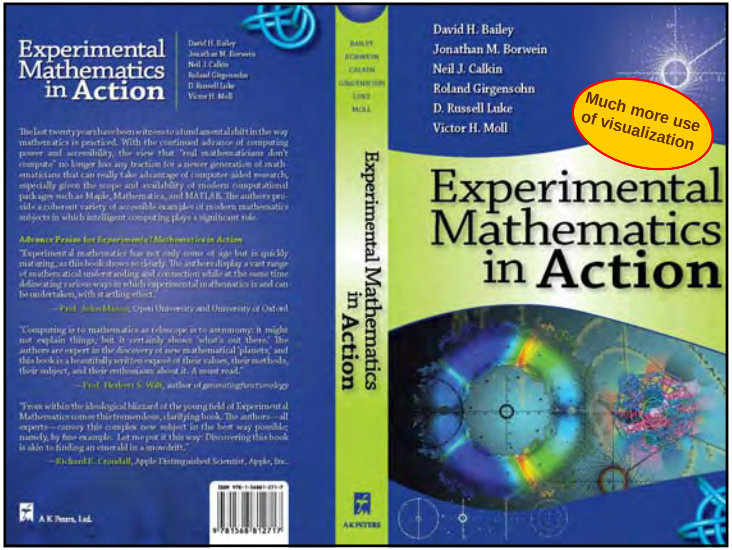
Experimental Mathodology

1. Gaining insight and intuition
2. Discovering new relationships
3. Visualizing math principles
4. Testing and especially falsifying conjectures
5. Exploring a possible result to see if it merits formal proof
6. Suggesting approaches for formal proof
7. Computing replacing lengthy hand derivations
8. Confirming analytically derived results



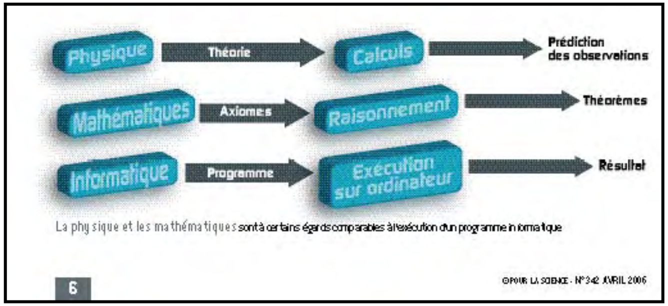


Comparing $-y^2\ln(y)$ (red) to $y-y^2$ and y^2-y^4



Math + Physics = Computing ?

- En français

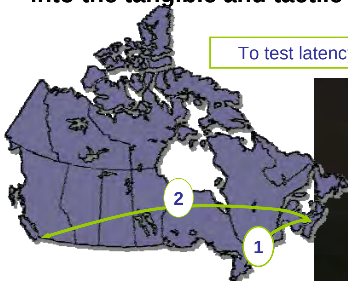


Haptics and Light Paths

D-DRIVE Doug a haptic mascot

Haptic Devices extend the world of I/O into the tangible and tactile

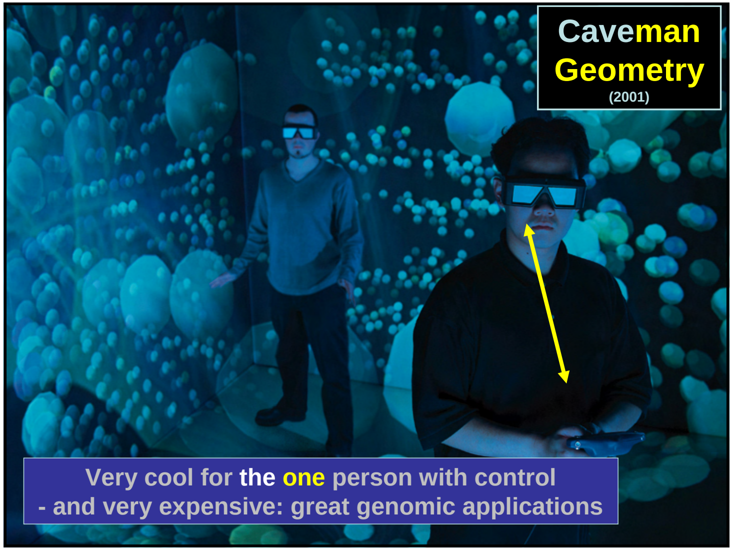
To test latency issues …



Links multiple devices so two or more users may interact at a distance (**BC/NS Demo** April 06)

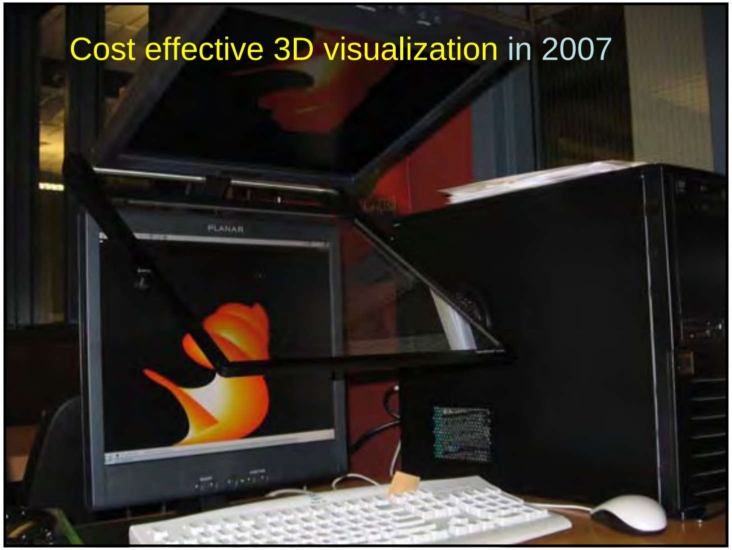
Sensable's **Phantom Omni**

- in Museums, **Aware Homes**, elsewhere
- Kinesiology, Surgery, Music, Art …



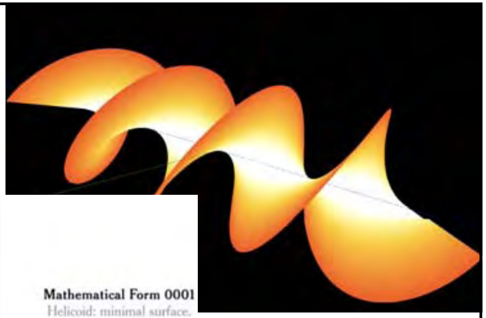
Caveman Geometry
(2001)

Very cool for the one person with control
- and very expensive: great genomic applications



Cost effective 3D visualization in 2007

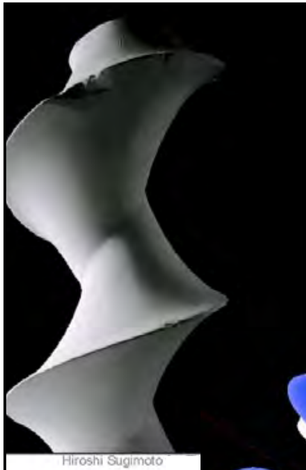
19th C model plus recent photograph and 21st C rendition



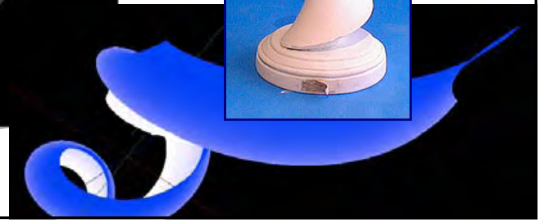
Mathematical Form 0001
Helicoid: minimal surface.

$$x = a \sinh v \cos u$$
$$y = a \sinh v \sin u$$
$$z = au$$
$$(0 \le u < 2\pi, -\infty < v < \infty)$$

19th C Plaster Model
Kline and Schwartz



Mathematical Form 0003



Coast to Coast ('C2C') Seminar

2008: will focus on PhD presentations
Chile has now joined

Lead partners:

Dalhousie D-Drive – Halifax Nova Scotia

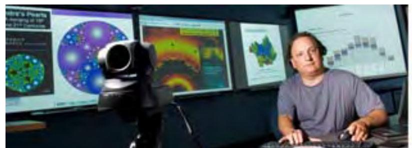
IRMACS – Burnaby, British Columbia

Other Participants so far include:

University of British Columbia, University of Alberta, University of Alberta, University of Saskatchewan, Lethbridge University, Acadia University, MUN, Mt Allison, St Francis Xavier University, University of Western Michigan, MathResources Inc, University of North Carolina, …

Tuesdays 3:30pm (Atlantic) 11.30am (Pacific)

✓Chapter in Communicating Mathematics in the Digital Era (AK Peters, Sept 2008)



I could be in Newcastle AG

CARMA is coming

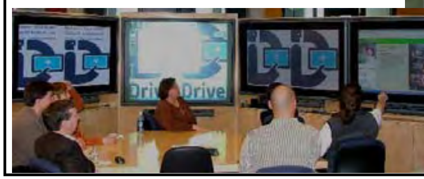
Computer **A**ssisted **R**esearch **M**aths and its **A**pplications

The Experience

Fully Interactive multi-way audio and visual interaction

Given good bandwidth audio is much harder

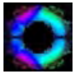
The closest thing to being in the same room



Shared Desktop for viewing presentations or sharing software

Content Dominates Form

High Quality Presentations



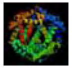
Jonathan Borwein, Dalhousie University
Mathematical Visualization

Uwe Glaesser, Simon Fraser University
Semantic Blueprints of Discrete Dynamic Systems

Peter Borwein, IRMACS
The Riemann Hypothesis

"No one explains chalk"

Jonathan Schaeffer, University of Alberta
Solving Checkers



Arvind Gupta, MITACS
The Protein Folding Problem

Przemyslaw Prusinkiewicz, University of Calgary
Computational Biology of Plants

Karl Dilcher, Dalhousie University
Fermat Numbers, Wieferich and Wilson Primes

Future Libraries will include very complex objects

III. New Ways of Seeing Math

- **The Colour Calculator**
  – numbers as pictures
- **The Inverse Calculator**
  – numbers go in and symbols come out
- **The Top Ten Numbers Website**

- All at http://ddrive.cs.dal.ca/~isc/portal

A Colour and an Inverse Calculator (1995 & **2007**)

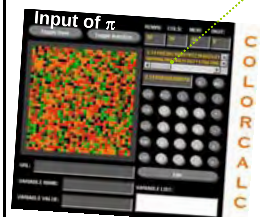
Nathalie Sinclair: **Aesthetic basis for middle-school maths** (2006)

Inverse Symbolic Computation

Archimedes: $223/71 < \pi < 22/7$

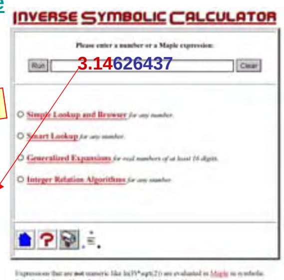
Inferring mathematical structure from numerical data

- Mixes *large table lookup*, integer relation methods and intelligent preprocessing – needs ***micro-parallelism***
- It faces the "curse of exponentiality"
- Implemented as **identify** in Maple



identify(sqrt(2.)+sqrt(3.))

$\sqrt{2}+\sqrt{3}$



Calculator (ISC) uses a combination of lookup tables and integer relation algorithms in order to associate with a user-defined, truncated decimal expansion (represented as a floating point expression) a closed form representation for the real number.

CDrive NSERC CRSNG Maplesoft

The ISC in Action

ISC+ inverse symbolic calculator

Standard lookup results for 12.587886229548403854

exp(1)+Pi^2

ISC The original ISC

The Dev Team: Nathan Singer, Andrew Shouldice, Lingyun Ye, Tomas Daske, Peter Dobcsanyi, Dante Manna, O-Yeat Chan, Jon Borwein

3.146264370 Try it!

19.99909998 Try it!

ISC The original ISC

The Dev Team: Nathan Singer, Andrew Shouldice, Lingyun Ye, Tomas Daske, Peter Dobcsanyi, Dante Manna, O-Yeat Chan, Jon Borwein

accepts either floating point expressions or correct Maple syntax as input. However, for Maple syntax requiring too long for evaluation, a timeout has been implemented.

Visit

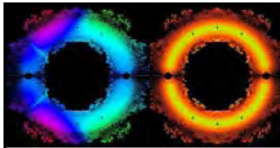
Jon Borwein's Webpage

David Bailey's Webpage

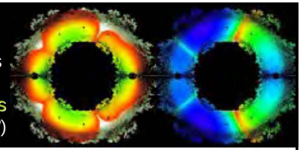
Math Resources Portal

- ISC+ runs on Glooscap
- Less lookup & more algorithms than 1995

Roots of Zeros



What you draw is what you see ("visible structures in number theory")



Striking fractal patterns formed by plotting complex zeros for all polynomials in powers of x with coefficients 1 and -1 to degree 18

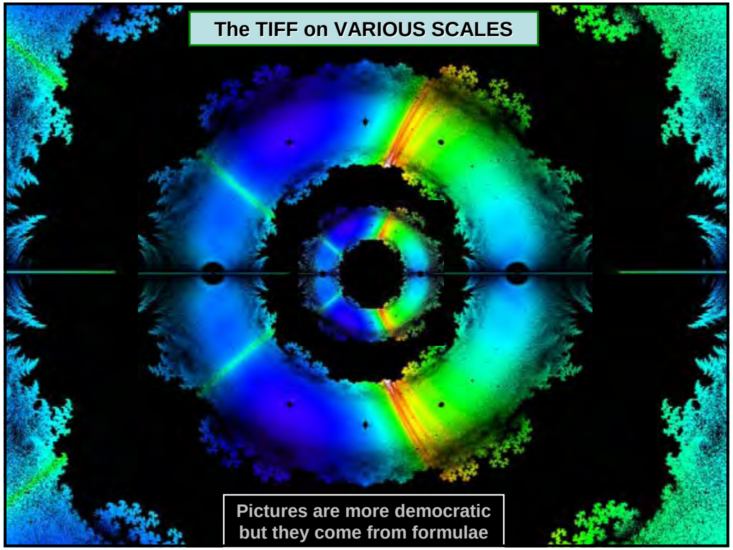
Coloration is by sensitivity of polynomials to slight variation around the values of the zeros. **The color scale represents a normalized sensitivity** to the range of values; red is insensitive to violet which is strongly sensitive.

- All zeros are pictured (at **3600 dpi**)
- Figure 1b is colored by their local density
- Figure 1d shows sensitivity relative to the x^9 term
- **The white and orange striations are not understood**

A wide variety of patterns and features become visible, leading researchers to totally unexpected mathematical results

"The idea that we could make biology mathematical, I think, perhaps is not working, but what is happening, strangely enough, is that maybe mathematics will become biological!" Greg Chaitin, Interview, 2000.

The TIFF on VARIOUS SCALES



Pictures are more democratic but they come from formulae

When is a Movie an Interactive Proof?

The Perko Pair 10_{161} and 10_{162}

are two adjacent 10-crossing knots (1900)

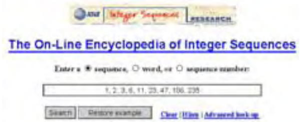
- **first shown to be the same by Ken Perko in 1974**
- **and beautifully made dynamic in KnotPlot (open source-ish)**

"What it comes down to is our software is too hard and our hardware is too soft."

IV. Amazing New Web Services

- **AT&T** Online Encyclopedia of Sequences

  What is 1,2,3,6,11,23,47,106,235,…?



Supernumerary Rainbow over Newton's birthplace

- **NIST** Digital Library of Math Functions

  What is an Airy Function?

MathResources Inc.

- **MAA** Digital Library with my company's free dictionary
  - also in *Maple* since 9.5

Soon the texts will also do lots of the maths

PlanetMath.Org MathDL Wolfram MathWorld

Greetings from the On-Line Encyclopedia of Integer Sequences!

Matches (up to a limit of 30) found for 1 2 3 6 11 23 47 106 235 :
[It may take a few minutes to search the whole database, depending on how many matches are found (the second and later look are faster)]

An Exemplary Database

```
ID Number: A000055 (Formerly M0791 and N0299)
URL:       http://www.research.att.com/projects/OEIS?Anum=A000055
Sequence:  1,1,1,1,2,3,6,11,23,47,106,235,551,1301,3159,7741,19320,
           48629,123867,317955,823065,2144505,5623756,14828074,
           39299897,104636890,279793450,751065460,2023443032,
           5469566585,14830871802,40330829030,109972410221
Name:      Number of trees with n unlabeled nodes.
Comments:  Also, number of unlabeled 2-gonal 2-trees with n 2-gons.
References F. Bergeron, G. Labelle and P. Leroux, Combinatorial Species and
             Tree-Like Structures, Camb. 1998, p. 279.
           N. L. Biggs et al., Graph Theory 1736-1936, Oxford, 1976, p. 49.
           S. R. Finch, Mathematical Constants, Cambridge, 2003, pp. 295-316.
           D. D. Grant, The stability index of graphs, pp. 29-52 of Combinatorial
             Mathematics (Proceedings 2nd Australian Conf.), Lect. Notes Math.
             403, 1974.
           F. Harary, Graph Theory. Addison-Wesley, Reading, MA, 1969, p. 232.
           F. Harary and E. M. Palmer, Graphical Enumeration, Academic Press, NY,
             1973, p. 58 and 244.
           D. E. Knuth, Fundamental Algorithms, 3d Ed. 1997, pp. 386-88.
           R. C. Read and R. J. Wilson, An Atlas of Graphs, Oxford, 1998.
           J. Riordan, An Introduction to Combinatorial Analysis, Wiley, 1958,
             p. 138.
Links:     P. J. Cameron, Sequences realized by oligomorphic permutation groups J. Integ. Seqs. Vo
           Steven Finch, Otter's Tree Enumeration Constants
           E. M. Rains and N. J. A. Sloane, On Cayley's Enumeration of Alkanes (or 4-Valent Trees)
           N. J. A. Sloane, Illustration of initial terms
           E. W. Weisstein, Link to a section of The World of Mathematics.
           Index entries for sequences related to trees
           Index entries for "core" sequences
           G. Labelle, C. Lamathe and P. Leroux, Labeled and unlabeled enumeration of k-gonal 2-tr
Formula:   G.f.: A(x) = 1 + T(x)-T^2(x)/2+T(x^2)/2, where T(x) = x + x^2 + 2*x^3 + ...
```

Integrated real time use

- moderated
- 142,759 entries
- grows daily
- AP book had 5,000

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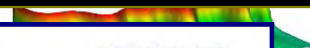
http://dlmf.nist.gov First 21C database



Digital Library of Mathematical Functions

Chapter AI. Airy & Related Functions
Properties

§AI.3. Graphs & Visualizations — §AI.5. Integral Representations

§AI.4. Maclaurin Series

For $z \in \mathbb{C}$

AI.4.1

$$Ai(z) = Ai(0)\left(1+\frac{1}{3!}z^3+\frac{1\cdot 4}{6!}z^6+\frac{1\cdot 4\cdot 7}{9!}z^9+\cdots\right)+Ai'(0)\left(z+\frac{2}{4!}z^4+\frac{2\cdot 5}{7!}z^7+\frac{2\cdot 5\cdot 8}{10!}z^{10}+\cdots\right)$$

Symbols used: AiryAi, cdots and z
A&S Ref: 10.4.2 (with 10.4.4 and 10.4.5)
Encodings: LaTeX
Parsed:
- AiryAi@(z) = AiryAi@(0) * (1 + (1 / 3!) * z ^ 3 + ((1 cdot 4) / 6!) * z ^ 6 + ((1 cdot 4 cdot 7) / 9!) * z ^ 9 + cdots) + (diffop@(AiryAi, 1))@(0) * (z + (2 / 4!) * z ^ 4 + ((2 cdot 5) / 7!) * z ^ 7 + ((2 cdot 5 cdot 8) / 10!) * z ^ 10 + cdots)

- Formula level metadata
- Mathematical searching
- Accessible output
- Latex, PNG, MathML

AI.4.2

$$Ai'(z) = Ai'(0)\left(1+\frac{2}{3!}z^3+\frac{2\cdot 5}{6!}z^6+\frac{2\cdot 5\cdot 8}{9!}z^9+\cdots\right)+Ai(0)\left(\frac{1}{2!}z^2+\frac{1\cdot 4}{5!}z^5+\frac{1\cdot 4\cdot 7}{8!}z^8+\cdots\right).$$

AI.4.3

$$Bi(z) = Bi(0)\left(1+\frac{1}{3!}z^3+\frac{1\cdot 4}{6!}z^6+\frac{1\cdot 4\cdot 7}{9!}z^9+\cdots\right)+Bi'(0)\left(z+\frac{2}{4!}z^4+\frac{2\cdot 5}{7!}z^7+\frac{2\cdot 5\cdot 8}{10!}z^{10}+\cdots\right)$$

AI.4.4

$$Bi'(z) = Bi'(0)\left(1+\frac{2}{3!}z^3+\frac{2\cdot 5}{6!}z^6+\frac{2\cdot 5\cdot 8}{9!}z^9+\cdots\right)+Bi(0)\left(\frac{1}{2!}z^2+\frac{1\cdot 4}{5!}z^5+\frac{1\cdot 4\cdot 7}{8!}z^8+\cdots\right)$$

Index
Notation
Search
Need Help?
About the Project
January 12, 2006
Supported by NIST National Institute of Standards and Technology and NSF

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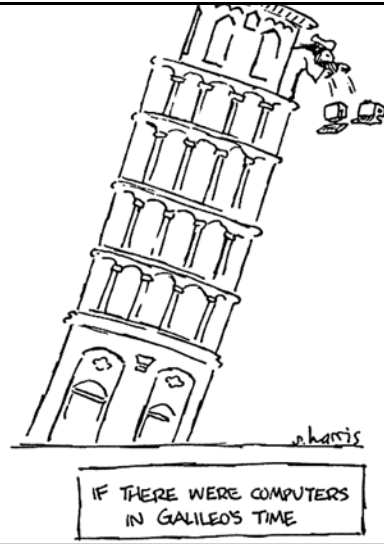
NIST National Institute of Standards and Technology

National Science Foundation Celebrating 50 Years

The faint line below the main colored arc is a `supernumerary rainbow', produced by the interference of different sun-rays traversing a raindrop and emerging in the same direction. For each color, the intensity profile across the rainbow is an Airy function. Airy invented his function in 1838 precisely to describe this phenomenon more accurately than Young had done in 1800 when pointing out that supernumerary rainbows require the wave theory of light and are impossible to explain with Newton's picture of light as a stream of independent corpuscles. The house in the picture is Newton's birthplace.



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Enigma

J.M. Borwein and D.H. Bailey, *Mathematics by Experiment: Plausible Reasoning in the 21st Century,* A.K. Peters, 2nd expanded edition, **2008**

and with

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"The object of mathematical rigor is to sanction and legitimize the conquests of intuition, and there was never any other object for it."

- J. Hadamard quoted at length in E. Borel, *Lecons sur la theorie des fonctions*, 1928.

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