



August 13th 2008

What's New, What's Possible, What's Coming ...



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"intuition comes to us much earlier and with much less outside influence than formal arguments which we cannot really understand unless we have reached a relatively high level of logical experience and sophistication."

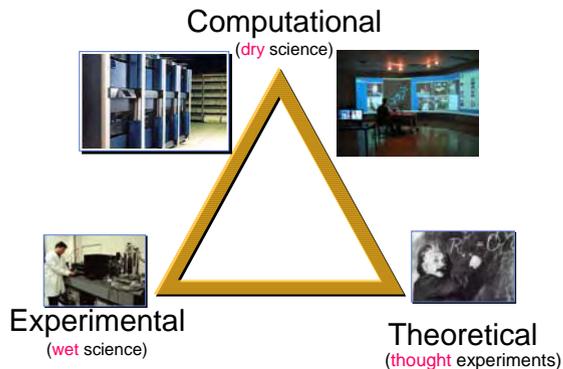


George Polya
1887-1985

Revised 13/08/2008



I. Changing Research Landscape: a new triangle



Outline of Presentation

- I. The **Changing Research** Landscape
- II. New Ways of **Doing Mathematics**
- III. New Ways of **Seeing Mathematics**
- IV. Amazing New **Web Services**

HPC Needs in Canada or Oz

2008 High Performance Computing Needs

The array of Canadian research projects each have unique high performance computing requirements.

- Ring 1** Desktop Computers / Small Clusters
- Ring 2** Mid-Range Systems (in the top 500 worldwide)
- Ring 3** Supercomputers / Terascale System (in the top 30 worldwide)



2008: a sustained Petaflop attained at LANL-- 2 years early

My Lab in Halifax

REYKJAVIK:4H
LONDON:4H
BERLIN:5H
TEL AVIV:6H
NEW DELHI:9.5H
CAPE TOWN:6H
WASHINGTON:1H
LOS ANGELES:4H
VANCOUVER:4H
TOKYO:11H
SYDNEY:10H
BRASILIA:1H
SANTIAGO:0H

within 5 hours difference

240 cpu Glooscap at Dal

D-Drive's Nova Scotia location lends us unusual freedom when interacting globally. Many cities around the world are close enough in a chronological sense to comfortably accommodate real-time collaboration.

Dalhousie Distributed Research Institute and Virtual Environment

Moore's Law and its Implications

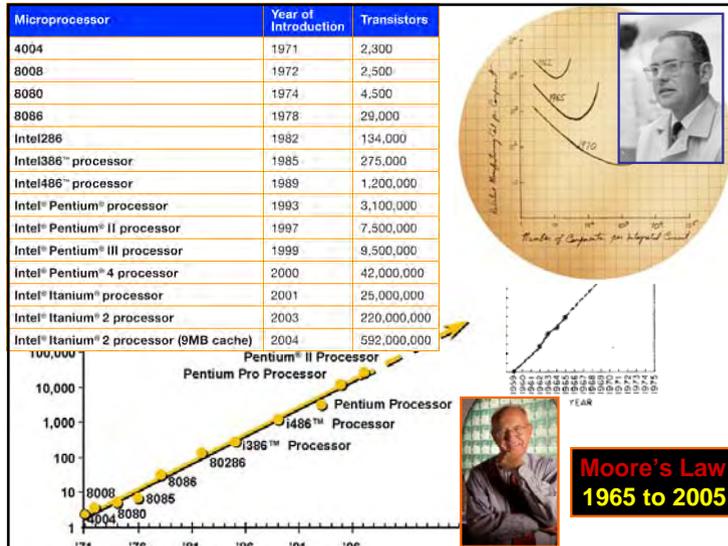
"The complexity for minimum component costs has increased at a rate of roughly a factor of two per year ...

- now taken as "every 18 months to 2 years"

Certainly over the short term this rate can be expected to continue, if not to increase. Over the longer term, the rate of increase is a bit more uncertain, although there is no reason to believe it will not remain nearly constant for at least 10 years. That means by 1975, the number of components per integrated circuit for minimum cost will be 65,000. I believe that such a large circuit can be built on a single wafer.

Gordon Moore (Intel) "Cramming more components onto [Electronic Circuits](#)", [Electronics Magazine 19 April 1965](#)

Unprecedented and expected to continue for 10-20 years.

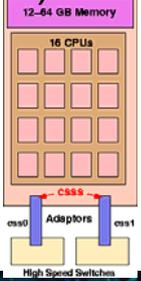


This picture is worth 100,000 ENIACs

The number of ENIACs needed to store the 20Mb TIF the Smithsonian sold me

1947 The past (5Kf/sec)

NERSC's 6000 cpu Seaborg in 2004 (10Tflops/sec)
 - we need new software paradigms for 'bigga-scale' hardware



The near-present

Mathematical Immersive Reality in Vancouver

IBM BlueGene/L at LANL

System
 (64 cabinets, 64x32x32)

IBM Computer Achieves Petaflop Performance

6/9/2008

A National Nuclear Security Administration (NNSA) supercomputer has achieved an operational rate of 1,000 trillion calculations per second, or 1 **petaflop**, making the Roadrunner -- which the NNSA commissioned IBM Corp. to build in 2006 for around \$130 million -- the world's fastest computer, the agency announced today.

2.8/5.6 GF/s
 4 MB

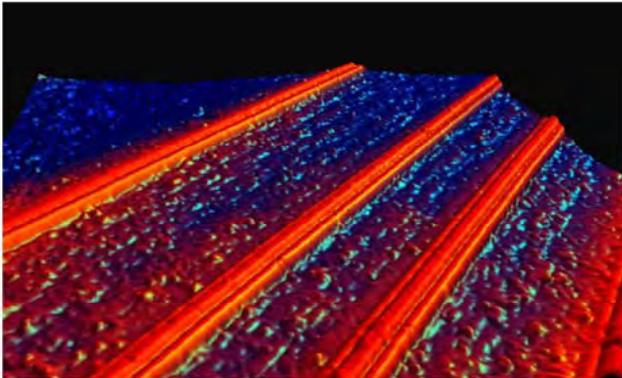
0.07/0.14 GF/s
 0.5 GB DDR

2¹⁷ cpu's: Oct 2007 ran Linpack benchmark at over 596 Tflop /sec
(5 x Canada or 8 x Oz)

The future 2005-2010

Things we can't model here include:

Self assembling wires 2nm apart (HP Labs)



The Changing Computing Landscape

Simulation by Keuckes-Williams



"It says it's sick of doing things like inventories and payrolls, and it wants to make some breakthroughs in astrophysics."

II. New Ways of Doing Math

- **and related subjects:** Computer Science, Statistics, Engineering, all Sciences, every other subject for learning or for research
 - Experimentally on the Computer
 - Visual or Haptic or Acoustic Output
 - Simulations and Emersions
 - With Web-services, Databases, Wikis, ...
 - Marvelous support tools for the classroom
- also **New Ways of Collaborating**

Jon Borwein's Math Resource Portal

The following is a list of useful math tools.



Utilities

1. [ISC2_0: The Inverse Symbolic Calculator](#)
2. [EZ Face : An interface for evaluation of Euler sums and Multiple Zeta Values](#)
3. [3D Function Grapher](#)
4. [GraPHedron: Automated and computer assisted conjectures in graph theory](#)
5. [Julia and Mandelbrot Set Explorer](#)
6. [Embree-Trefethen-Wright pseudospectra and eigenproblem](#)

Reference

7. [The On-Line Encyclopedia of Integer Sequences](#)
8. [Finch's Mathematical Constants](#)
9. [The Digital Library of Mathematical Functions](#)
10. [The Prime Pages](#)

Content

11. [Experimental Mathematics Website](#)
12. [Wolfram Mathworld](#)
13. [Planet Math](#)
14. [Numbers, Constants, and Computation](#)
15. [Wikipedia: Mathematics](#)

ICCOPT 2007 Short Course

16. [Jon's Lectures](#)

Experimental Methodology

1. Gaining **insight** and intuition
2. Discovering new relationships
3. **Visualizing** math principles
4. Testing and especially **falsifying conjectures**
5. Exploring a possible result to see **if it merits formal proof**
6. Suggesting approaches for formal proof
7. Computing replacing lengthy hand derivations
8. Confirming analytically derived results

MATH LAB
Computer experiments are transforming mathematics
BY BRUCE ALLENBERG
Science News 2004

Many people regard mathematics as the crown jewel of the sciences. Yet math has lately fully taken on the role of the leading edge of science. Modern theories of physics, biology, and chemistry are being tested and refined through computer experiments. In fact, many of the most important discoveries in these fields have been made through computer experiments.

Mathematicians, by contrast, continue to do their work in a more traditional way. They use computers to help them understand abstract concepts, but they do not use them to discover new results. This is changing. The advent of powerful computers and the development of new mathematical tools have made it possible to explore complex mathematical problems in ways that were previously impossible.

One example of this is the study of fractals. Fractals are complex, self-similar shapes that can be found in nature and in art. They are a key part of chaos theory and have many applications in science and engineering. Computer experiments have allowed mathematicians to explore the properties of fractals in ways that were previously impossible.

Another example is the study of optimization problems. These are problems where the goal is to find the best solution among a large number of possible solutions. Computer experiments have allowed mathematicians to explore these problems in ways that were previously impossible.

Finally, computer experiments have allowed mathematicians to explore the properties of complex systems. These are systems where the behavior of the whole is not simply the sum of the parts. Computer experiments have allowed mathematicians to explore these systems in ways that were previously impossible.

Comparing $-y^2 \ln(y)$ (red) to y^2 and $y^2 - y^4$

Experimental Mathematics in Action

David H. Bailey
Jonathan M. Borwein
Neil J. Calkin
Roland Girsengohn
D. Russell Luke
Victor H. Mall

The last twenty years have borne witness to a fundamental shift in the way mathematics is practiced. With the continual advance of computing power and accessibility, the view that "real mathematics is done by computer" no longer has any traction for a mature generation of mathematicians that can truly take advantage of computer aided research, especially given the speed and availability of modern computational tasks such as Maple, Mathematica, and MATLAB. The authors provide a coherent variety of accessible examples of modern mathematic subjects in which intelligent computing plays a significant role.

Advance Praise for Experimental Mathematics in Action

"Experimental mathematics has not only come of age but is quickly maturing, and this book shows it clearly. The authors display a vast range of mathematical understanding and connection while at the same time delineating various ways in which experimental mathematics is and can be undertaken, with an ending error."

—Prof. Adam Marcus, Open University and University of Oxford

"Computing is to mathematics as telescope is to astronomy: it might not explain things, but it certainly shows what's out there. The authors are expert in the discovery of new mathematical 'planets', and this book is a beautiful written report of their findings, their methods, their subjects, and their enthusiasm about it. A must read."

—Prof. Eckhard S. Wehr, author of *geneticinformatics*

"From within the ideological hinterland of the young field of Experimental Mathematics comes this tremendous, clarifying book. The authors—all experts—convey their complex new subject in the best way possible, aided by fine examples. Let me put it this way: 'Discovering' this book is akin to finding an oasis in a snowdrift."

—Richard E. Crandall, Apple Entomorphical Scientist, Apple, Inc.

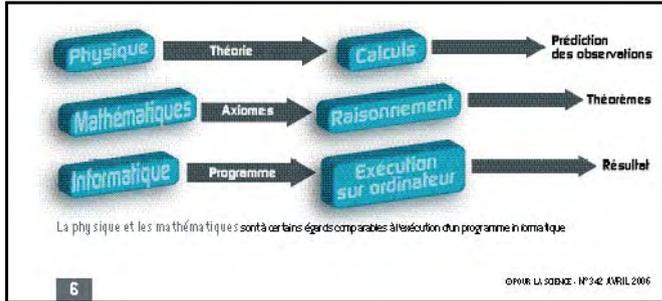
Experimental Mathematics in Action

Much more use of visualization

A K Peters, Inc.

Math + Physics = Computing ?

- En français



Haptics and Light Paths

D-DRIVE Doug a haptic mascot

Haptic Devices extend the world of I/O into the tangible and tactile



To test latency issues ...



Links multiple devices so two or more users may interact at a distance (BC/NS Demo April 06)

- in Museums, Aware Homes, elsewhere
- Kinesiology, Surgery, Music, Art ...

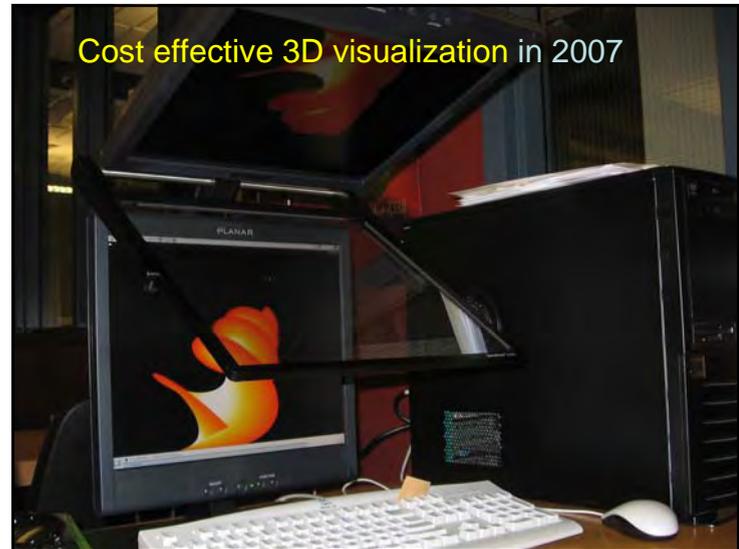
Sensable's Phantom Omni

Caveman Geometry

(2001)

Very cool for the **one** person with control
- and very expensive: great genomic applications

Cost effective 3D visualization in 2007

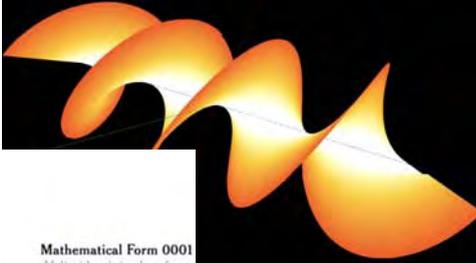


19th C model
 plus recent
 photograph and
 21st C rendition



Mathematical Form 0001
 Helicoid: minimal surface.

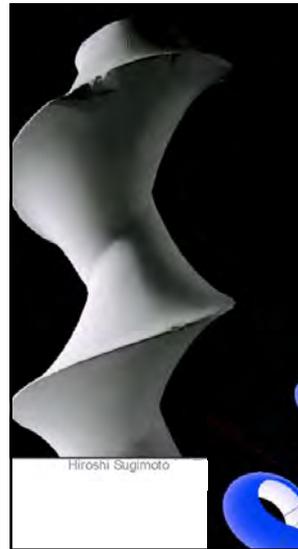
$$\begin{aligned} x &= a \sinh v \cos u \\ y &= a \sinh v \sin u \\ z &= au \\ (0 \leq u < 2\pi, -\infty < v < \infty) \end{aligned}$$



Hiroshi Sugimoto for The New York Times

19th C Plaster Model
[Kline and Schwartz](#)

Mathematical Form 0003



Hiroshi Sugimoto

z
 $(0 \leq z$



Dalhousie Distributed Research Institute and Virtual Environment

Coast to Coast ("C2C") Seminar

2008: will focus on
 PhD presentations
 Chile has now joined

Lead partners:

Dalhousie D-Drive – Halifax
 Nova Scotia

IRMACS – Burnaby,
 British Columbia

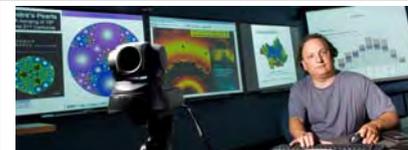
Other Participants so far include:

University of British Columbia, University
 of Alberta, University of Alberta, University
 of Saskatchewan, Lethbridge University,
 Acadia University, MUN, Mt Allison, St
 Francis Xavier University, University of
 Western Michigan, MathResources Inc,
 University of North Carolina, ...



Tuesdays 3:30pm (Atlantic) T1.30am (Pacific)

✓ Chapter in [Communicating Mathematics in the Digital Era](#) (AK Peters, Sept 2008)



I could be in Newcastle AG
CARMA is coming
 Computer Assisted Research Maths
 and its Applications

The Experience

Fully Interactive multi-way audio
 and visual interaction

Given good bandwidth audio is
 much harder

The closest thing to being in the
 same room



Shared Desktop for
 viewing presentations or
 sharing software

Content Dominates Form



Jonathan Borwein, Dalhousie University
Mathematical Visualization

High Quality Presentations

Uwe Glaesser, Simon Fraser University
Semantic Blueprints of Discrete Dynamic Systems



Peter Borwein, IRMACS
The Riemann Hypothesis

"No one explains chalk"

Jonathan Schaeffer, University of Alberta



Arvind Gupta, MITACS
The Protein Folding Problem

Solving Checkers



Przemyslaw Prusinkiewicz, University of Calgary
Computational Biology of Plants



Karl Dilcher, Dalhousie University
Fermat Numbers, Wieferich and Wilson Primes

Future Libraries will include
very complex objects



"Solving Checkers"

Speaker in
Edmonton

Audience in
Vancouver

April 2007 [Checkers solved](#)

[Science](#): one of top 10
break-throughs of 2007

2006: [Poincaré Conjecture](#)
top breakthrough of year

III. New Ways of Seeing Math

- The Colour Calculator
 - numbers as pictures
- The Inverse Calculator
 - numbers go in and symbols come out
- The Top Ten Numbers Website



- All at <http://ddrive.cs.dal.ca/~isc/portal>

A Colour and an Inverse
Calculator (1995 & 2007)

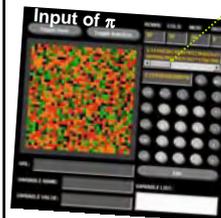
Nathalie Sinclair: Aesthetic
basis for middle-school maths
(2006)

Inverse Symbolic Computation

Archimedes: $223/71 < \pi < 227/71$

Inferring mathematical structure from numerical data

- Mixes *large table lookup*, integer relation methods and intelligent preprocessing – needs *micro-parallelism*
- It faces the "curse of exponentiality"
- Implemented as **identify** in **Maple**



identify(sqrt(2)+sqrt(3..))

$\sqrt{2} + \sqrt{3}$

INVERSE SYMBOLIC CALCULATOR



Expressions that are not constants like $\ln(3)^{\sqrt{2}}$ are evaluated in Maple to machine precision. © Copyright by a private user and published by a teacher.

Calculator (ISC) uses a combination of lookup tables and integer relation algorithms in order to associate with a user-defined, truncated decimal expansion (represented as a floating point expression) a closed form representation for the real number.

Logo Drive NSERC CRNG MapleSoft

The ISC in Action

isc+ inverse symbolic calculator

Standard lookup results for 12.587886229548403854

$\exp(1)+\pi^2$

ISC The original ISC

The Dev Team: Nathan Singer, Andrew Shouldice, Lingyun Ye, Tomas Daske, Peter Dobcsanyi, Dante Manna, O-Yeet Chan, Jon Borwein

Visit

Jon Borwein's Webpage

David Bailey's Webpage

Math Resources Portal

3.146264370 Try it!

19.99909998 Try it!

ISC The original ISC

The Dev Team: Nathan Singer, Andrew Shouldice, Lingyun Ye, Tomas Daske, Peter Dobcsanyi, Dante Manna, O-Yeet Chan, Jon Borwein

accepts either floating point expressions or correct Maple syntax as input. However, for Maple syntax requiring too long for evaluation, a timeout has been implemented.

ISC+ runs on Gloscap

Less lookup & more algorithms than 1995

Roots of Zeros

What you draw is what you see ("visible structures in number theory")

Striking fractal patterns formed by plotting complex zeros for all polynomials in powers of x with coefficients 1 and -1 to degree 18

Coloration is by sensitivity of polynomials to slight variation around the values of the zeros. **The color scale represents a normalized sensitivity** to the range of values; red is insensitive to violet which is strongly sensitive.

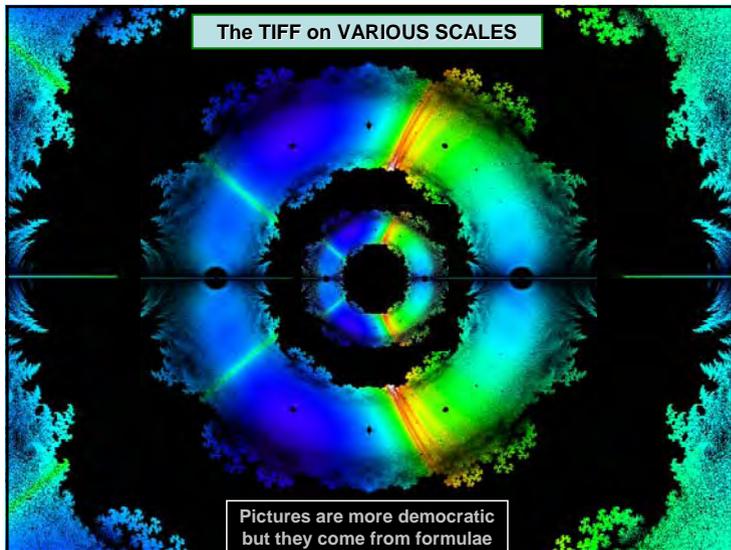
- All zeros are pictured (at 3600 dpi)
- Figure 1b is colored by their local density
- Figure 1d shows sensitivity relative to the x^9 term
- The white and orange striations are not understood**

A wide variety of patterns and features become visible, leading researchers to totally unexpected mathematical results

"The idea that we could make biology mathematical, I think, perhaps is not working, but what is happening, strangely enough, is that maybe mathematics will become biological!"

Greg Chaitin, [Interview](#), 2000.

The TIFF on VARIOUS SCALES



Pictures are more democratic but they come from formulae

When is a Movie an Interactive Proof?

The Perko Pair 10_{161} and 10_{162}

are two adjacent 10-crossing knots (1900)



- first shown to be the same by Ken Perko in 1974
- and beautifully made dynamic in [KnotPlot](#) (open source-ish)



"What it comes down to is our software is too hard and our hardware is too soft."

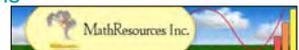
IV. Amazing New Web Services

- **AT&T Online Encyclopedia of Sequences**
What is 1,2,3,6,11,23,47,106,235,...



Supernumerary Rainbow over Newton's birthplace

- **NIST Digital Library of Math Functions**
What is an Airy Function?



- **MAA Digital Library** with my company's **free dictionary**
– also in *Maple* since 9.5

Soon the texts will also do lots of the maths



Greetings from the On-Line Encyclopedia of Integer Sequences!

Matches (up to a limit of 50) found for: 1 2 3 6 11 23 47 106 235:

[It may take a few minutes to search the whole database, depending on how many matches are found (the second and later lookups are faster)]

An Exemplary Database

ID Number: A000055 (Formerly N0791 and N0299)
URL: <http://www.research.att.com/projects/OEIS?num=A000055>
Sequence: 1, 1, 1, 1, 2, 7, 6, 11, 23, 47, 106, 235, 551, 1301, 3159, 7741, 19220, 49529, 123267, 317955, 821645, 2144525, 5482756, 14033074, 39299897, 104636890, 279791450, 751054540, 2023443032, 548954585, 14810871802, 40210829030, 109972410221

Number of trees with a unlabeled nodes.

References: F. Bergeron, G. Labelle and P. Lecout, Combinatorial Species and Tree-Like Structures, Camb. 1998, p. 279.
N. L. Biggs et al., Graph Theory 1736-1936, Oxford, 1976, p. 49.
S. R. Finch, Mathematical Constants, Cambridge, 2003, pp. 198-216.
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R. C. Read and R. J. Wilson, An Atlas of Graphs, Oxford, 1998.
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Links: P. J. Cameron, Sequences realized by oligomorphic permutation groups J. Teleg. Seqs. Vol. Steven Finlay, [Order's Tree Enumeration Constants](#)
E. M. Ryskin and W. A. S. Siano, [On Cayley's Enumeration of Alkanes \(or s-Valent Trees\)](#)
N. J. A. Sloane, [Illustration of initial terms](#)
E. M. Weisstein, [link to a section of The World of Mathematics](#).
Index entries for sequences related to trees
Index entries for "catalan" sequence
G. Labelle, C. Lamathe and P. Lecout, Labeled and unlabeled enumeration of k-quasi-1-trees.
G. Z.: $A(x) = 1 + T(x) - T^2(x) + T^2(x)^2/2$, where $T(x) = x + x^2 + 2x^3 + \dots$

Integrated real time use

- moderated
- 142,759 entries
- grows daily
- AP book had 5,000

<http://dlmf.nist.gov> First 21C database

Digital Library of Mathematical Functions

Chapter 4: Airy & Related Functions

§4.1.4. Maclaurin Series

For $z \in \mathbb{C}$

AI.4.1

$$Ai(z) = Ai(0) \left(1 + \frac{1}{3!}z^3 + \frac{1}{6!}z^6 + \frac{1}{9!}z^9 + \dots \right) + Ai'(0) \left(z + \frac{1}{4!}z^4 + \frac{1}{7!}z^7 + \frac{1}{10!}z^{10} + \dots \right)$$

• Formula level metadata

• Accessible output

• LaTeX, PNG, MathML

Need Help?

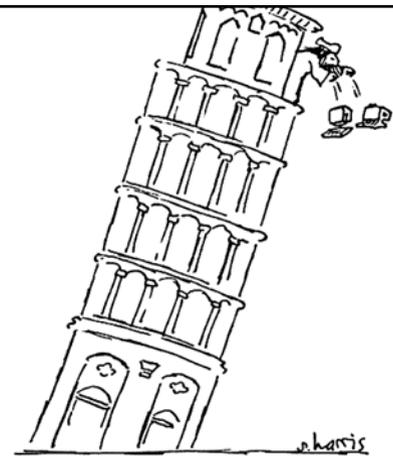
Supported by NIST National Institute of Standards and Technology and National Institute of Mathematical and Biological Sciences

- Index
- Notations
- Search
- Need Help?
- Customize
- Show Annotations

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The faint line below the main colored arc is a 'supernumerary rainbow', produced by the interference of different sun-rays traversing a raindrop and emerging in the same direction. For each color, the intensity profile across the rainbow is an Airy function. Airy invented his function in 1838 precisely to describe this phenomenon more accurately than Young had done in 1800 when pointing out that supernumerary rainbows require the wave theory of light and are impossible to explain with Newton's picture of light as a stream of independent corpuscles. The house in the picture is Newton's birthplace.



IF THERE WERE COMPUTERS
IN GALILEO'S TIME

REFERENCES



Dalhousie Distributed Research Institute and Virtual Environment



Enigma

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and with

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D.H. Bailey and J.M. Borwein, "Experimental Mathematics: Examples, Methods and Implications," *Notices AMW*, 52 No. 5 (2005), 502-514.

J. Borwein, D. Bailey, N. Calkin, R. Girgensohn, R. Luke, and V. Moll, *Experimental Mathematics in Action*, A.K. Peters, 2007.

Jon Borwein and Keith Devlin, *The Computer as Crucible*, A.K. Peters, November, 2008.

"The object of mathematical rigor is to sanction and legitimize the conquests of intuition, and there was never any other object for it."

- J. Hadamard quoted at length in E. Borel, *Lecons sur la theorie des fonctions*, 1928.

Karl Heinz Hoffmann's Cover Illustrations for **The Computer as Crucible**

