Who we are and how we got that way

Revised for MAA volume on **The Mind of a Mathematician**

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ABSTRACT. The typical research mathematician's view of the external world's view of mathematicians is more pessimistic and less nuanced than any objective measure would support. I shall explore some of the reasons why I think this is so. I submit that mathematics is a "science of the artificial" [18] and that we should wholeheartedly embrace such a positioning of our subject.

1 Putting Things in Perspective

> All professions look bad in the movies ... why should scientists expect to be treated differently? —Michael Crichton[1]

I greatly enjoyed Steve Krantz's article in this collection that he showed me when I asked him to elaborate what he had in mind. I guess I am less pessimistic than he is. This may well reflect the different milieus we have occupied. I see the same glass but it is half full.

Some years ago, my brother Peter surveyed other academic disciplines. He discovered that students who complain mightily about calculus professors still prefer the relative certainty of what we teach and assess to the subjectivity of a creative writing course or the rigors of a physics or chemistry laboratory course. Similarly, while I have met my share of micro-managing Deans—who view mathematics with disdain when they look at the size of our research grants or the infrequency of our patents—I have encountered more obstacles to mathematical innovation within than without the discipline.

I do wish to aim my scattered reflections in generally the right direction: I am more interested in issues of creativity á la Hadamard [4] than in Russell and foundations or Piaget and epistemology... and I should like a dash of "goodwill computing" thrown in.

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[1] Addressing the 1999 AAAS Meetings, as quoted in *Science* of Feb. 19, 1999, p.1111.

More seriously, I wish to muse about how we work, what keeps us going, how the mathematics profession has changed and how "*la plus ca change, la plus ca reste la même*",[2] and the like while juxtaposing how we perceive these matters and how we are perceived. Elsewhere, I have discussed at length my own views about the nature of mathematics from both an aesthetic and a philosophical perspective (see, e.g., [10, 19]). I have described my self as 'a computer-assisted quasi-empiricist'. For present more psychological proposes I will quote approvingly from [5, p. 239]:

> ... Like 0l' Man River, mathematics just keeps rolling along and produces at an accelerating rate "*200,000 mathematical theorems of the traditional handcrafted variety ... annually.*" Although sometimes proofs can be mistaken—sometimes spectacularly—and it is a matter of contention as to what exactly a "proof" is—there is absolutely no doubt that the bulk of this output is correct (though probably uninteresting) mathematics.— Richard C. Brown

Why do we produce so many unneeded results? In addition to the obvious pressure to publish and to have something to present at the next conference, I suspect Irving Biederman's observations below plays a significant role.

> While you're trying to understand a difficult theorem, it's not fun," said Biederman, professor of neuroscience in the USC College of Letters, Arts and Sciences. ... "But once you get it, you just feel fabulous." ... The brain's craving for a fix motivates humans to maximize the rate at which they absorb knowledge, he said. ... "I think we're exquisitely tuned to this as if we're junkies, second by second."—Irving Biederman[3]

Take away all success or any positive reinforcement and most mathematicians will happily replace research by adminstration, more and (hopefully better) teaching, or perhaps just a favourite hobby. But given just a little stroking by colleagues or referees and the occasional opiate jolt, and the river rolls on.

The pressure to publish is unlikely to abate and qualitative measurements of performance[4] are for the most part fairer than leaving everything to the whim of one's Head of Department. Thirty years ago my career review consisted of a two-line mimeo "*your salary for next year will be ...*" with the relevant number written in by hand. At the same time, it is a great shame that mathematicians have a hard time finding funds to go to conferences just to listen and interact. Csikszentmihalyi [6] writes:

> [C]reativity results from the interaction of a system composed of three elements: a culture that contains symbolic rules, a person who brings novelty into the symbolic domain, and a field of experts who recognize and validate the innovation. All three are necessary for a creative idea, product, or discovery to take place.—Mihalyy Csikszentmihalyi

[2]For an excellent account of the triumphs and vicissitudes of Oxford mathematics over eight centuries see [8]. The description of Haley's ease in acquiring equipment (telescopes) and how he dealt with inadequate money for personnel is by itself worth the price of the book.,

[3]Discussing his article in the *American Scientist* at `www.physorg.com/news70030587.html`

[4]For an incisive analysis of citation metrics in mathematics I thoroughly recommend the recent IMU report and responses at: `http://openaccess.eprints.org/index.php?/archives/417-Citation-Statistics-International-Mathematical-Union-Report.html`.

We have not paid enough attention to what creativity is and how it is nurtured. Conferences need audiences and researchers need feedback other than the mandatory "*nice talk*" at the end of a special session. We have all heard distinguished colleagues mutter a stream of criticism during a plenary lecture only to proffer "*I really enjoyed that*" as they pass the lecturer on the way out. A communal view of creativity requires more of the audience.

2 Who We Are

As to who we are? Sometimes we sit firmly and comfortably in the sciences. Sometimes we practice—as the *Economist* noted—the most inaccessible of the arts[5] possessed in Russell's terms [17, p. 60] of *"a supreme beauty—a beauty cold and austere."* And sometimes we sit or feel we sit entirely alone. So forgive me if my categorizations slip and slide a bit. Even when we wish to remove ourselves from the sciences—by dint perhaps of our firm deductive underpinnings—they are often more than welcoming. They largely fail to see the stark deductive/inductive and realist/idealist distinctions which reached their apogee in the past century.

Yet many scientists have strong mathematical backgrounds. A few years ago I had the opportunity to participate as one of a team of seven scientists and one humanist who were mandated to write a national report on Canada's future need for advanced computing [14]. Five of us had at least an honours degree in mathematics. At the time none of us (myself included) lived in a mathematics department. The human genome project, the burgeoning development of financial mathematics, finite element modeling, Google and much else have secured the role of mathematics within modern science and technology research and development as "*the language of high technology*"; the most sophisticated language humanity has ever developed. Indeed, in part this scientific ecumenism reflects what one of my colleagues has called "*an astonishing lack of appreciation for how mathematics is done.*" He went on to remark that in this matter we are closer to the fine arts.

Whenever I have worked on major interdisciplinary committees, my strong sense has been of the substantial respect and slight sense of intimidation that most other quantitative scientists have for mathematics. I was sitting on a multi-science national panel when Wiles' proof of Fermat's last theorem was announced. My confreres wanted to know "*What, why and how?*" 'What' was easy, as always 'why' less so, and I did not attempt 'how'. In [10] I wrote

> While we mathematicians have often separated ourselves from the sciences, they have tended to be more ecumenical. For example, a recent review of *Models. The Third Dimension of Science*[6] chose a mathematical plaster model of a Clebsch diagonal surface as its only illustration. Similarly, authors seeking examples of the aesthetic in science often choose iconic mathematics formulae such as $\mathrm{E} = \mathrm{MC}^2$.

[5]In "Proof and Beauty," *Economist* article, 31 Mar 2005. "*Why should the non-mathematician care about things of this nature? The foremost reason is that mathematics is beautiful, even if it is, sadly, more inaccessible than other forms of art.*"

[6]See Julie K. Brown, "Solid Tools for Visualizing Science," *Science,* November 19, 2004, 1136–37.

'How' is not easy even within mathematics. *A Passion for Science* [21] is the written record of thirteen fascinating BBC interviews with scientists including Nobelist Abdus Salam, Stephen Jay Gould, Michael Berry and Christopher Zeeman. The communalities of their scientific experiences far outstrip the differences. Zeeman tells a nice story of how his Centre's administrator (a non mathematician) in Warwick could tell whether the upcoming summer was dedicated to geometry and topology, to algebra, or to analysis—purely on the basis of their domestic arrangements and logistics. For instance algebraists were very precise in their travel plans, topologists very inclusive in their social group activities and analysts were predictably unpredictable. I won't spoil the anecdote entirely but it reinforces my sense that the cognitive differences between those three main divisions of pure mathematics are at least as great as those with many cognate fields. In this taxonomy I am definitely an analyst not a geometer or an algebraist.

There do appear to be some cognitive communalities across mathematics. In [4] my brother with Peter Liljedahl and Helen Zhai report on the responses to an updated version of Hadamard's questionnaire [13] which they circulated to a cross-section of leading living mathematicians. This was clearly a subject the target group wanted to speak about. The response rate was excellent (over 50%) and the answers striking. According to the survey responses, the respondents placed a high premium on serendipity—but as Pasteur observed "*fate favours the prepared mind.*" Judging by where they said they have their best ideas they take frequent showers and like to walk while thinking. They don't read much mathematics, preferring to have mathematics explained to them in person. They much more resemble theorists throughout the sciences than careful methodical scholars in the humanities.

My academic life started in the short but wonderful infusion of resources for science and mathematics 'after sputnik'—I started University in 1967— and now includes the *Kindle Reader* (on which I am listening[7] to a fascinating new biography of the *Defense Advanced Research Projects Agency*, DARPA). The tyranny of a Bourbaki-dominated curriculum has been largely replaced by the scary grey-literature world of *Wikipedia* and *Google scholar*.

While typing this paragraph I went out on the web and found the Irving Layton poem, that I quote at the start of Section 4, in entirety within seconds (I merely googled "*And me happiest when I compose poems*" (I know the poem is *somewhere* in my personal library). For the most part this has been a wonderful journey. Not everything has improved from that halcyon pre-post-structuralist period a half-century ago when algebraists could command more attention from funding agencies than could engineers as [5] recalls. But the sense of time for introspection before answering a colleague's wafer-thin 'airmail letter' enquiry, and the smell of mold that accompanied leisurely rummaging in a great library's stacks are losses in my personal life measure the receding role of the University as the "*last successful medieval institution.*"[11]

2.1 Stereotypes from without looking in

One of the epochal events of my childhood as a faculty brat in St. Andrews, Scotland was when C. P. Snow (1905–1980) delivered an immediately controversial 1959 Rede

[7] They will read to you in a friendly if unnatural voice.

Lecture in Cambridge entitled "The Two Cultures".[8] Snow argued that the breakdown of communication between the "two cultures" of modern society—the sciences and the humanities—was a major obstacle to solving the world's problems—and he had never heard of global warming. In particular, he noted the quality of education was everywhere on the decline. Instancing that many scientists had never read Dickens, while those in the humanities were equally non-conversant with science, he wrote:

> A good many times I have been present at gatherings of people who, by the standards of the traditional culture, are thought highly educated and who have with considerable gusto been expressing their incredulity at the illiteracy of scientists. Once or twice I have been provoked and have asked the company how many of them could describe the Second Law of Thermodynamics, the law of entropy. The response was cold: it was also negative. Yet I was asking something which is about the scientific equivalent of: *'Have you read a work of Shakespeare's?'*

The British musical satirists Michael Flanders and Donald Swann took immediate heed of this for their terrific monologue and song "First and Second Law of Thermodynamics" that I can still recite from memory.

> [Michael:] Snow says that nobody can consider themselves educated who doesn't know at least the basic language of Science. I mean, things like Sir Edward Boyle's Law, for example: the greater the external pressure, the greater the volume of hot air. Or the Second Law of Thermodynamics - this is very important. I was somewhat shocked the other day to discover that my partner not only doesn't know the Second Law, he doesn't even know the First Law of Thermodynamics.
>
> Going back to first principles, very briefly, thermodynamics is of course derived from two Greek words: *thermos*, meaning hot, if you don't drop it, and *dinamiks*, meaning dynamic, work; and thermodynamics is simply the science of heat and work and the relationships between the two, as laid down in the Laws of Thermodynamics, which may be expressed in the following simple terms...
>
> After me...
>
> The First Law of Thermodymamics:
> Heat is work and work is heat
> Heat is work and work is heat
> Very good!
> The Second Law of Thermodymamics:
> Heat cannot of itself pass from one body to a hotter body
> (scat music starts)
> Heat cannot of itself pass from one body to a hotter body
> Heat won't pass from a cooler to a hotter
> Heat won't pass from a cooler to a hotter
> You can try it if you like but you far better notter
> You can try it if you like but you far better notter
> 'Cos the cold in the cooler will get hotter as a ruler
>
> ...

[8] Subsequently republished in [20].

Snow goes on to say:

> I now believe that if I had asked an even simpler question - such as, *What do you mean by mass, or acceleration*, which is the scientific equivalent of saying, *'Can you read?'* - not more than one in ten of the highly educated would have felt that I was speaking the same language. So the great edifice of modern physics goes up, and the majority of the cleverest people in the western world have about as much insight into it as their Neolithic ancestors would have had.

C. P. Snow wrote pre-Kuhn, pre-Foucault, pre-much else [5]; and I submit that a half-century on the situation is worse, knowledge more fragmented, ignorance of science and mathematics more damaging to the public discourse.[9]

In addition, I think the problem was much less symmetric than Snow suggested. I doubt I have ever met a scientist who had not read (or at least watched on BBC) some Dickens, who never went to movies, art galleries or the theatre. It is, however, ever more socially acceptable to be a scientific ignoramus or a mathematical dunce. It is largely allowed to boast "*I was never any good at mathematics at school.*" I was once told exactly that—in soto voce—by the then Canadian Governor General during a formal ceremony at his official residence in Ottawa. Even here we should be heedful not to over-analyse as we are prone to do. Afterwards the 'GG' (as Canadians call their Queen's designate) ruminated apologetically that if he had been a bit better at mathematics he would not have had to become a journalist. Some of this has been 'legitimated' by denigrating science as 'reductionist' and incapable of the deeper verities [5].

As Underwood Dudley has commented, no one apologizes for not being good at geology in school. Most folks understand that failing "Introduction to Rocks" in Grade Nine does not knock you off of a good career path. The outside world knows several truths: mathematics is important, it is hard, it is usually poorly taught in school, and the average middle-class parent is ill-prepared to redress the matter. I have become quite hard-line about this. When a traveling companion on a plane starts telling me that "*Mathematics was my worst subject in school.*" I will reply "*And if you were illiterate would you tell me*?" They usually take the riposte fairly gracefully.

Consider two currently popular TV dramas *Numb3rs* (mathematical) and *House* (medical). A few years ago a then colleague, a distinguished pediatrician, asked me whether I watched Num3rs. I replied "*Do you watch House? Does it sometimes make you cringe?*" He admitted that it did but he still watched it. I said the same was true for me with Numb3rs, that my wife loved it and that I liked lots about it. It made mathematics seem important and was rarely completely off base. The lead character, Charlie, was brilliant and good-looking with a cute smart girl friend. The resident space-cadet on the show was a physicist not a mathematician. What more could one ask for? Sadly for many of our

[9] I can't resist including the following email anecdote:

> This morning Al Gore gave the "keynote" speech at SC09.[10] During the question-answer period, he mentioned a famous talk "The Two Cultures" about lack of communication between science and humanities, by one Chester (??)—he drew a blank as to who it was. Sitting on the third row, I shouted out "Snow" (meaning C. P. Snow). One other person also shouted "Snow", and so Gore acknowledged that it was indeed Snow.

.

colleagues the answer is "*absolute fidelity to mathematical truth in every jot and title.*" No wonder so many of us make a dog's-breakfast of the opportunities given to publicize our work!

> 'Caution, skepticism, scorn, distrust and entitlement seem to be intrinsic to many of us because of our training as scientists.—Stephen Rosen[11]

To "*Caution, skepticism, scorn, distrust and entitlement,*" I'd add "*persistence, intensity, a touch of paranoia, and a certain lack of sartorial elegance*" but I still would not have identified mathematicians within the larger scientific herd. I think we are more inward drawn than theorists in, say, biology or physics. Our terminologies are more speciated between subfields and so we typically graze in smaller groups. But we are still bona fide scientists—contrary to the views of some laboratory scientists and some of our own colleagues.

> This is the essence of science. Even though I do not understand quantum mechanics or the nerve cell membrane, I trust those who do. Most scientists are quite ignorant about most sciences but all use a shared grammar that allows them to recognize their craft when they see it. The motto of the Royal Society of London is 'Nullius in verba' : trust not in words. Observation and experiment are what count, not opinion and introspection. Few working scientists have much respect for those who try to interpret nature in metaphysical terms. For most wearers of white coats, philosophy is to science as pornography is to sex: it is cheaper, easier, and some people seem, bafflingly, to prefer it. Outside of psychology it plays almost no part in the functions of the research machine.—Steve Jones[12]

2.2 Stereotypes from within looking out

Philosophy (not to mention introspection) is arguably more important to, though little more respected by, working mathematicians than it is to experimental scientists.

> Whether we scientists are inspired, bored, or infuriated by philosophy, all our theorizing and experimentation depends on particular philosophical background assumptions. This hidden influence is an acute embarrassment to many researchers, and it is therefore not often acknowledged. Such fundamental notions as reality, space, time, and causality–notions found at the core of the scientific enterprise–all rely on particular metaphysical assumptions about the world. —Christof Koch[13]

As I alluded to above, working mathematicians—by which I mean those of my personal or professional acquaintance—are overinclined by temperament and training to see meaning where none is intended and patterns where none exist. For the most part over

[11]An astrophysicist, turned director of the *Scientific Career Transitions Program* in New York City, giving job-hunting advice in an on-line career counseling session as quoted in *Science*, August 4 1995, p. 637. He continues that these traits hinder career change!

[12]From his review of "How the Mind Works" by Steve Pinker, in the *New York Review of Books*, pp. 13-14, Nov 6, 1997.

[13]In "Thinking About the Conscious Mind," a review of John R. Searle's *Mind. A Brief Introduction*, Oxford University Press, 2004.

the past centuries this somewhat autistic tendency has been a positive adaptation. It has allowed the discipline to develop the most powerful tools and most sophisticated descriptive language possessed by mankind. But as the nature of mathematics changes we should be heedful of Napoleon's adage "*Never ascribe to malice that which is adequately explained by incompetence,*"[14] or as Goethe (1749-1832) put it in [12]:

> Misunderstandings and neglect occasion more mischief in the world than even malice and wickedness. At all events, the two latter are of less frequent occurrence.

Suppose for 'malice/wickedness' we substitute 'meaning/reason' and likewise replace 'incompetence/misunderstandings' by 'chance/randomness'. Then these squibs provide an important caution against seeing mathematical patterns where none exist. They offer equally good advice when dealing with Deans.

3 Changing Modes of Doing Mathematics

Goethe's advice is especially timely as we enter an era of intensive computer-assisted mathematical data-mining; an era in which we will more-and-more encounter unprovable truths and salacious falsehoods. In [10] I wrote

> It is certainly rarer to find a mathematician under thirty who is unfamiliar with at least one of *Maple*, *Mathematica* or MATLAB, than it is to find one over sixty five who is really fluent. As such fluency becomes ubiquitous, I expect a re-balancing of our community's valuing of deductive proof over inductive knowledge.

As we again become comfortable with mathematical *discovery* in Giaquinto's sense of being "*independent, reliable and rational*" [9], assisted by computers, the community sense of a mathematician as a producer of theorems will probably diminish to be replaced by a richer community sense of mathematical understanding. It has been said that Riemann proved very few theorems and even fewer correctly and yet he is inarguably one of the most important mathematical, indeed scientific, thinkers of all time. Similarly most of us were warned off pictorial reasoning:

> A heavy warning used to be given [by lecturers] that pictures are not rigorous; this has never had its bluff called and has permanently frightened its victims into playing for safety. Some pictures, of course, are not rigorous, but I should say most are (and I use them whenever possible myself).—J. E. Littlewood [16, p. 53][15]

Let me indicate how much one can now do with good computer-generated pictures.

3.1 Discovery and proof: Divide-and-concur

In a wide variety of problems such as protein folding, 3SAT, spin glasses, giant Sodoku, etc., we wish to find a point in the intersection of two sets A and B where B is non-convex.

[14] I have collected variants old and new on the theme of over-ratiocination at `www.carma.newcastle.edu/jb616/quotations.html`

[15] Littlewood (1885-1977) published this in 1953 and so long before the current fine graphic, geometric, and other visualization tools were available.

The notion of "divide-and-concur" as described below often works spectacularly—much better than theory can currently explain. Let $P_A(x)$ and $R_A(x) := 2\,P_A(x) - x$ denote respectively the *projector* and *reflector* on a set A as illustrated in Figure 1. Then "divide-and-concur"[16] is the natural geometric iteration "reflect-reflect-average":

$$x_{n+1} \Longrightarrow \frac{x_n + R_A\left(R_B(x_n)\right)}{2}. \tag{1}$$

Consider the simplest case of a line A of height α and the unit circle B [2]. With $z_n := (x_n, y_n)$ we have:

$$x_{n+1} := \cos\theta_n, y_{n+1} := y_n + \alpha - \sin\theta_n, \quad (\theta_n := \arg z_n). \tag{2}$$

This is intended to find a point on the intersection of the unit circle and the line of height α as shown in Figure 2 for $\alpha = .94$.

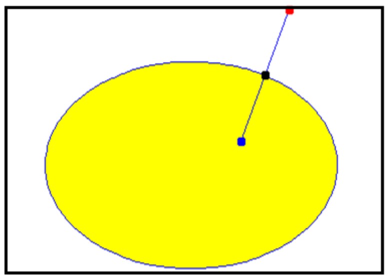

Figure 1: Reflector (interior) and Projector (boundary) of a point external to an ellipse.

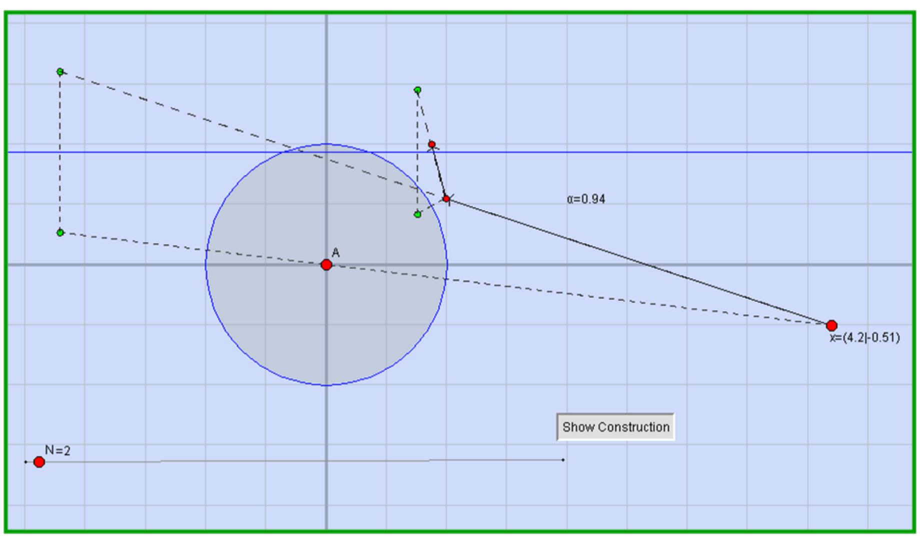

Figure 2: The first three iterates of (2) in *Cinderella*.

[16]This is Cornell physicist Veit Elser's slick term for the algorithm in which the reflection can be performed on separate cpu's (divide) and then averaged (concur).

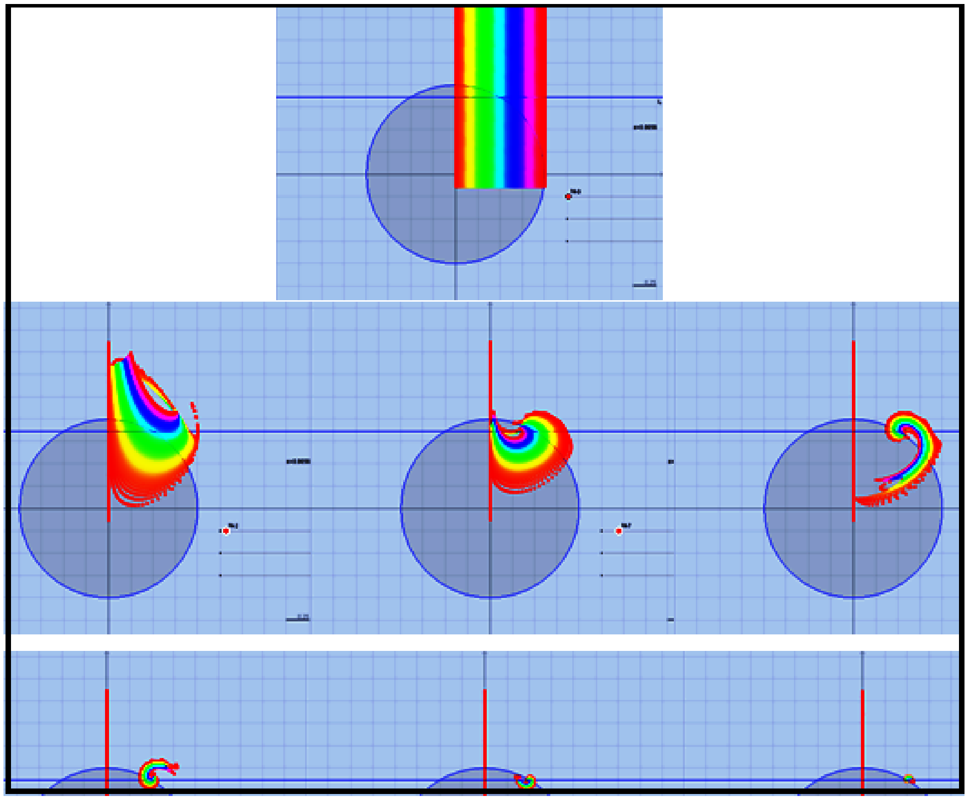

Figure 3: Snapshots of 10, 000 points after 0, 2, 7, 13, 16, 21, and 27 steps of (2).

We have also studied the analogous differential equation since asymptotic techniques for such differential equations are better developed. We decided

$$
\begin{aligned}
(3) \qquad x'(t) &= \frac{x(t)}{r(t)} - x(t) \text{ where } r(t) := \sqrt{x(t)^2 + y(t)^2}\,] \\
y'(t) &= \alpha - \frac{y(t)}{r(t)}
\end{aligned}
$$

was a reasonable counterpart to the Cartesian formulation of (2)—we have replaced the difference $x_{n+1} - x_n$ by $x'(t)$, etc.—as shown in Figure 4.

Following Littlewood, I find it hard to persuade myself that the pictures in Figures 3 and 4 do not constitute a *generic proof* of the algorithms they display as implemented in an applet at `http://users.cs.dal.ca/~jborwein/expansion.html`. In Figure 3 we see the iterates spiralling in towards the right-hand point of intersection with those closest to the y-axis lagging behind but being unremittingly reeled in to the point. Brailey Sims and I have now found a conventional proof that the behaviour is as observed [3] but we discovered all the results first graphically and were lead to the appropriate proofs by the dynamic pictures we drew.

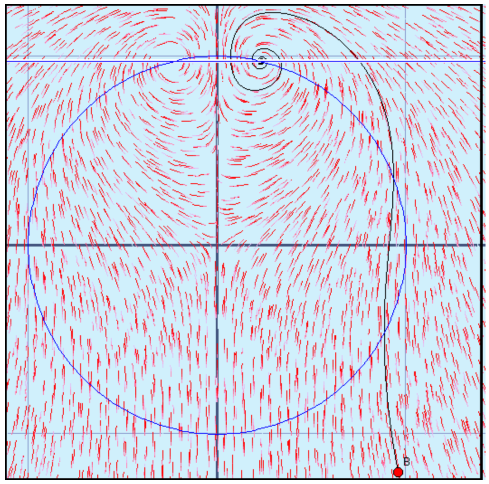

Figure 4: ODE solution and vector field for (3) with $\alpha = 0.97$ in *Cinderella*.

4 The Exceptionalism of Mathematics

> And me happiest when I compose poems.
> Love, power, the huzza of battle
> Are something, are much;
> yet a poem includes them like a pool.—Irving Layton [15, p. 189]

This is the first stanza of the Irving Layton (1912-2006) poem "The Birth of Tragedy." Explicitly named after Nietzsche's first book, Layton tussles with Apollonian and Dionysian impulses (reason versus emotion). He calls himself "A quiet madman, never far from tears" and ends "*while someone from afar off blows birthday candles for the world.*" Layton, who was far from a recluse, is one of my favourite Canadian poets.

I often think poetry is a far better sustained metaphor for mathematics than either music or the plastic arts. I do not see poetry making such a good marriage with any other science. Like good poets, good mathematicians are often slightly autistic observers of a somewhat dysphoric universe. Both art forms at their best distill and concentrate beauty like no other and both rely on a delicate balance of form and content, semantics and syntax.

Like all academic disciplines we are (over-)sure of our own specialness.

- *Mathematicians are machines for turning coffee into theorems.* (Renyi)
- *A gregarious mathematician is one who looks at the other person's feet when addressing them.*
- *Mathematics is what mathematicians do late at night.*
- *You want proof. I'll give you proof.* (Harris)
- *There are three kinds of mathematician, those who can count and those who can't.*

Most of these can be—and many have been— used with a word changed here or there about statisticians, computer scientists, chemists, physicists, economists and philosophers.

For instance "*There are 10 kinds of computer scientists, those who understand binary and those who don't.*" It is amusing to ask colleagues in other sciences for their corresponding self-identifying traits. All of the above mentioned groups except the philosophers are pretty much reductionists:

> Harvard evolutionary psychologist Steven Pinker is probed on "*Evolutionary Psychology and the Blank Slate.*" The conversation moves from the structure of the brain to adaptive explanations for music, creationism, and beyond. Stangroom asks Pinker about the accusations that biological explanations of behavior are determinist and reduce human beings to the status of automatons."..."*Most people have no idea what they mean when they level the accusation of determinism,*" "Pinker answers. "*It's a nonspecific "boo" word, intended to make something seem bad without any content.*[17]

Steve Jones is quoted in the same article equating philosophy and pornography and while many of us, myself included, see a current need to rethink the philosophy of mathematics, Pinker and he capture much of the zeitgeist of current science including mathematics.

4.1 Mathematics as a science of the artificial

Pure mathematics, theoretical computer science, and various cognate disciplines are sciences of the artificial in that they study *scientifically* man-made *artificial* concepts. Mathematical experiments and data collection are clearly not taking place in the natural world. They are at best quasi-empirical and yet they subscribe fully to the scientific method. Like other sciences they are increasingly engaged in "exploratory experimentation" [1, 2]. In *The Sciences of the Artificial* [18, p. 16] Herb Simon compellingly wrote about reductionism:

> This skyhook-skyscraper construction of science from the roof down to the yet unconstructed foundations was possible because the behaviour of the system at each level depended only on a very approximate, simplified, abstracted characterization at the level beneath.[1] This is lucky, else the safety of bridges and airplanes might depend on the correctness of the "Eightfold Way" of looking at elementary particles.
>
>> [1] "More than fifty years ago Bertrand Russell made the same point about the architecture of mathematics. See the "Preface" to Principia Mathematica "*... the chief reason in favour of any theory on the principles of mathematics must always be inductive, i.e., it must lie in the fact that the theory in question allows us to deduce ordinary mathematics. In mathematics, the greatest degree of self-evidence is usually not to be found quite at the beginning, but at some later point; hence the early deductions, until they reach this point, give reason rather for believing the premises because true consequences follow from them, than for believing the consequences because they follow from the premises.*"

[17] *The Scientist* of June 20, 2005 describing Jeremy Stangroom's interviews in *What (some) scientists say*, Routledge Press, 2005.

> Contemporary preferences for deductive formalisms frequently blind us to this important fact, which is no less true today than it was in 1910.

I love the fact that Russell the arch-deductivist so clearly describes the fundamental role of inductive reasoning within mathematics. This long-but-rewarding quote leads me to reflect that we mathematicians need more strong-minded and assured critics. I acknowledge that it is easier to challenge a speaker in history or philosophy. One may reasonably disagree in a way that is hard in mathematics.[18] When someone stands up in a mathematics lecture and says she can answer the speaker's hard open question, nine times out of ten the respondent has misunderstood the question or misremembered her own prior work. We do, however, need to develop a culture which encourages spirited debate of such matters as how best to situate our subject within the academy, how important certain areas and approaches are, how to balance research and scholarship, and so on. Moreover, fear and lack of mutual respect for another's discipline makes it hard to venture outside one's own niche. For instance, many physicists fear mathematicians who, in turn, are often most uncomfortable or dismissive of informal reasoning and of 'physical or economic intuition.'

4.2 Pure versus applied mathematics

Mathematics is at once both a set of indispensable tools and a self-motivating discipline; a mind-set and a way of thinking. In consequence there are many research mathematicians working outside mathematics departments and a smaller but still considerable number of non-mathematicians working within. What are the consequences? First, it is no longer possible to assume that all of one's colleagues could in principle—if not with enthusiasm or insight—teach all the mathematics courses in the first two years of the university syllabus. This pushes us in the direction of other disciplines like history or biology in which teaching has always been tightly coupled with core research competence.

At a more fundamental level, I see the discipline boundary as being best determined by answering the question as to whether the mathematics at issue is worth doing in its own right. If the answer is "yes", then it is 'pure'[19] mathematics and belongs in the discipline; if not then, however useful or important the outcome, it does not fit. The later would, for example, be the case of a lot of applied operations research, a good deal of numerical modeling and scientific computation, and most of statistics. All significant mathematics should be nourished within mathematics departments, but there are many important and useful applications that do not by that measure belong.

5 How to Become a Grownup Science

As Darwin [7] ruefully realized rather late in life, we mathematicians have a lot to offer:

[18] Some years ago I persuaded Amazon to remove several unsubstantiated assertions about "errors on every page" in one of my books—by a digital groupy turned stalker—from their website after I pointed out that while one could have an opinion that a Cormac McCarthy novel was dull but assertions of factual error were subject to test.

[19] Which may well be highly applicable.

> During the three years which I spent at Cambridge my time was wasted, as far as the academical studies were concerned, as completely as at Edinburgh and at school. I attempted mathematics, and even went during the summer of 1828 with a private tutor (a very dull man) to Barmouth, but I got on very slowly. The work was repugnant to me, chiefly from my not being able to see any meaning in the early steps in algebra. This impatience was very foolish, and in after years I have deeply regretted that I did not proceed far enough at least to understand something of the great leading principles of mathematics, for men thus endowed seem to have an extra sense.—Charles Darwin

We also have a lot to catch up with. We have too few accolades compared to other sciences: prize lectures, medals, fellowships and the like. We are insufficiently adept at boosting our own cases for tenure, for promotion or for prizes. We are frequently too honest in reference letters. We are often disgracefully terse—unaware of the need to make obvious to others what is for us blindingly obvious. I have seen a Fields medalist recommend a talented colleague for promotion with the one line letter "*Anne has done some quite interesting work.*" Leaving aside the ambiguity of the use of the word "quite" when sent by a European currently based in the United States to a North American promotion committee, this summary is pretty lame when compared to a three page letter for an astrophysicist or chemist—that almost always tells you the candidate is the top whatever-it-is in the field. A little more immodesty in promoting our successes is in order.

I'm not encouraging dishonesty, but it is necessary to understand the ground rules of the enterprise and to make some attempt to adjust to them. When a good candidate for a Rhodes Scholarship turns up at ones office, it should be obvious that a pro forma scrawled note

> Johnny is really smart and got an 'A+' in my advanced algebraic number theory class. You should give him a Rhodes scholarship.

is inadequate. Sadly, the only letters of that kind that I've seen in Rhodes scholarship dossiers have come from mathematicians.

I am a mathematician rather than a computational scientist or a computer scientist primarily because I savour the structures and curiosities (including *spandrels* and *exaptations* in Gould's words) of mathematics. I am never satisfied with my first proof of a result and until I have found limiting counter-examples and adequate corollaries will continue to worry at it. I like attractive generalizations on their own merits. Very often it is the unexpected and unintended consequences of a mathematical argument that when teased out provides the real breakthrough. Such often leads eventually to tangible and dramatic physical consequences: take quantum mechanical tunneling.

A few years ago I had finished a fine piece of work with a frequent collaborator who is a quantum field theorist—and a man of great insight and mathematical power. We had met success by introducing a sixth-root of unity into our considerations. I mooted looking at higher-order analogues. The reply came back "*God in her wisdom is happy to build the universe with sixth-roots. You, a mathematician, can look for generalizations if you wish.*"

6 Conclusion

I became a mathematician largely because mathematics satisfied four criteria. (i) I found it reasonably easy; (ii) I liked understanding or working out how things function; but (iii) I was not much good with my hands and had limited physical intuition; (iv) I really disliked pipettes but I wanted to be a scientist. That left mathematics. Artificial yes, somewhat introspective yes, but informed by many disciplines and clearly an important science.

I have had several students whom I can not imagine following any other life path but I was not one of those. I would I imagine have been happily fulfilled in various careers of the mind; say as an historian or an academic lawyer. But I became a mathematician. It has been and continues to be a pretty wonderful life.

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