CARMA and Me
or
What am I doing in Oz?

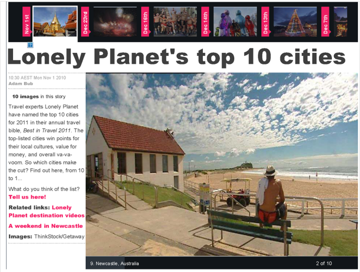
Jonathan M. Borwein FRSC FAA FAAAS

Laureate Professor & Director of CARMA, University of Newcastle
URL: http://carma.newcastle.edu.au/jon/carma-fest.pdf

Priority Research Centre
for
Computer **A**ssisted **R**esearch **M**athematics and its **A**pplications

Revised: July 16, 2011

Australia for Dummies and Wildlife Lovers


Lonely Planet's top 10 cities

10:30 AEST Mon Nov 1 2010
Adam Bub

10 images in this story

Travel experts Lonely Planet have named the top 10 cities for 2011 in their annual travel bible, *Best in Travel 2011*. The top-listed cities win points for their local cultures, value for money, and overall va-va-voom. So which cities make the cut? Find out here, from 10 to 1...

What do you think of the list? **Tell us here!**

Related links: Lonely Planet destination videos

A weekend in Newcastle

Images: ThinkStock/Getaway

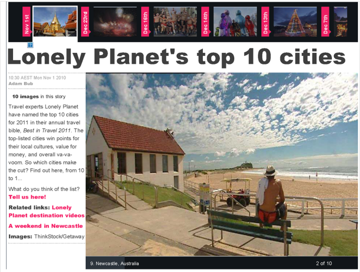
9. Newcastle, Australia — 2 of 10

⇐ Top 10 Places to See in 2011

Great ⇓ Wine ⇓ Water and ⇑ Beaches

CARMA

Australia for Dummies and Wildlife Lovers


Nov 1st Dec 23rd Dec 16th Dec 14th Dec 13th Dec 7th

Lonely Planet's top 10 cities

10:30 AEST Mon Nov 1 2010
Adam Bub

10 images in this story

Travel experts Lonely Planet have named the top 10 cities for 2011 in their annual travel bible, *Best in Travel 2011*. The top-listed cities win points for their local cultures, value for money, and overall va-va-voom. So which cities make the cut? Find out here, from 10 to 1...

What do you think of the list? **Tell us here!**

Related links: Lonely Planet destination videos

A weekend in Newcastle

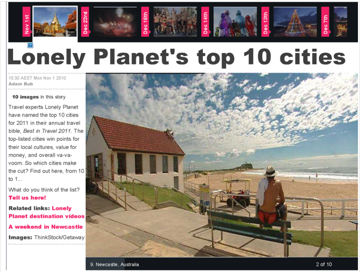
Images: ThinkStock/Getaway

9. Newcastle, Australia 2 of 10

⇐ Top 10 Places to See in 2011

Great ⇓ Wine ⇓ Water and ⇑ Beaches

Australia for Dummies and Wildlife Lovers



Nov 1st · Dec 23rd · Dec 16th · Dec 14th · Dec 13th · Dec 7th

Lonely Planet's top 10 cities

10:30 AEST Mon Nov 1 2010
Adam Bub

10 images in this story

Travel experts Lonely Planet have named the top 10 cities for 2011 in their annual travel bible, *Best in Travel 2011*. The top-listed cities win points for their local cultures, value for money, and overall va-va-voom. So which cities make the cut? Find out here, from 10 to 1...

What do you think of the list? **Tell us here!**

Related links: Lonely Planet destination videos

A weekend in Newcastle

Images: ThinkStock/Getaway

9. Newcastle, Australia — 2 of 10

$\Leftarrow$ Top 10 Places to See in 2011

Great $\Downarrow$ Wine $\Downarrow$ Water and $\Uparrow$ Beaches

CARMA

Priority Research Centre for Computer-Assisted Research Mathematics and its Applications

Search CARMA

- THE DIRECTOR'S WEBPAGES
- CARMA MEMBERS and VISITORS
- CARMA EVENTS
- CARMA NEWS
- CARMA RESEARCH
- RELATED LINKS AND SITES
- CONTACT US

MEMBER LINKS: (LOGIN REQUIRED)

- CARMA WIKI
- CARMA MEDIA COLLECTION

Subscribe to our seminar mailing list

CARMA OUTREACH | CARMA JOBS | RELATED LINKS AND SITES | CARMA PUBLICATIONS | ABOUT CARMA

UPCOMING EVENTS

CARMA RETREAT
[Conference Room, 412 Sandgate Road, Shortland (Hunter Wetlands Centre)]

CARMA Retreat 2011

- **Location:** Conference Room, 412 Sandgate Road, Shortland (Hunter Wetlands Centre)
- **Dates:** Tue, 19th Jul 2011 - Tue, 19th Jul 2011

CARMA COLLOQUIUM
[V129, Mathematics Building]

- **Speaker:** Boris Mordukhovich, Department of Mathematics, Wayne State University
- **Title:** *Generalized Newton's method based on graphical derivatives*
- **Location:** V129, Mathematics Building
- **Time and Date:** 4:00 pm, Thu, 21st Jul 2011

CARMA COLLOQUIUM
[V129, Mathematics Building]

- **Speaker:** Prof David Bailey, Lawrence Berkeley National Laboratory
- **Title:** *Hand-to-Hand Combat with Thousand-Digit Integrals*
- **Location:** V129, Mathematics Building
- **Time and Date:** 12:00 pm, Fri, 22nd Jul 2011

CARMA SEMINAR
[V129, Mathematics Building]

- **Speaker:** Wojciech Kozlowski, University of NSW
- **Title:** *Common fixed points for semigroups of pointwise Lipschitzian mappings in Banach spaces*
- **Location:** V129, Mathematics Building
- **Time and Date:** 2:00 pm, Fri, 22nd Jul 2011

See also about CARMA events external lectures

NEWS HIGHLIGHTS

CARMA RETREAT:

CARMA Retreat at the Hunter Wetlands Centre, Shortland, on Tuesday, 19th July, 2011. [Retreat Agenda]

Approbation for Applied Maths:

Numbers add up for Newcastle research: Newcastle applied maths best in Australia. [Article]

School Math Resources:

Online math resources for students [link]

CARMA News

Events for the Week

Contents. We will *sample* the following:

Experimental Mathematics: what it is?

Experimental mathematics is the use of a computer to run computations — sometimes no more than trial-and-error tests — to look for patterns, to identify particular numbers and sequences, to gather evidence in support of specific mathematical assertions that may themselves arise by computational means, including search.

Like contemporary chemists — and before them the alchemists of old — who mix various substances together in a crucible and heat them to a high temperature to see what happens, today's experimental mathematicians put a hopefully potent mix of numbers, formulas, and algorithms into a computer in the hope that something of interest emerges. (JMB-Devlin, 2008, p. 1)

- Quoted in International Council on Mathematical Instruction *Study 19: On Proof and Proving*, 2011.

CARMA

Experimental Mathematics: what it is?

Experimental mathematics is the use of a computer to run computations — sometimes no more than trial-and-error tests — to look for patterns, to identify particular numbers and sequences, to gather evidence in support of specific mathematical assertions that may themselves arise by computational means, including search.
Like contemporary chemists — and before them the alchemists of old — who mix various substances together in a crucible and heat them to a high temperature to see what happens, today's experimental mathematicians put a hopefully potent mix of numbers, formulas, and algorithms into a computer in the hope that something of interest emerges. (JMB-Devlin, 2008, p. 1)

- Quoted in International Council on Mathematical Instruction *Study 19: On Proof and Proving*, 2011.

CARMA

Experimental Mathematics: Integer Relation Methods

Secure Knowledge without Proof. Given real numbers $\beta, \alpha_1, \alpha_2, \ldots, \alpha_n$ Ferguson's integer relation method (PSLQ), finds a nontrivial linear relation of the form

$$a_0\beta + a_1\alpha_1 + a_2\alpha_2 + \cdots + a_n\alpha_n = 0, \qquad (1)$$

where a_i are integers — if one exists and provides an exclusion bound otherwise.

- If $a_0 \neq 0$ then (1) assures β is in rational vector space generated by $\{\alpha_1, \alpha_2, \ldots, \alpha_n\}$.
- $\beta = 1, \alpha_i = \alpha^i$ means α is algebraic of degree n.
- In **2000** *Computing in Science and Engineering* named PSLQ one of the top 10 algorithms of the 20th century.

PROFILE: HELAMAN FERGUSON

Carving His Own Unique Niche, In Symbols and Stone

By refusing to choose between mathematics and art, a self-described "misfit" has found the place where parallel careers meet

CMS D.Borwein Prize

Madelung constant

Experimental Mathematics: Integer Relation Methods

Secure Knowledge without Proof. Given real numbers $\beta, \alpha_1, \alpha_2, \ldots, \alpha_n$ Ferguson's integer relation method (PSLQ), finds a nontrivial linear relation of the form

$$a_0\beta + a_1\alpha_1 + a_2\alpha_2 + \cdots + a_n\alpha_n = 0, \qquad (1)$$

where a_i are integers — if one exists and provides an exclusion bound otherwise.

- If $a_0 \neq 0$ then (1) assures β is in rational vector space generated by $\{\alpha_1, \alpha_2, \ldots, \alpha_n\}$.
- $\beta = 1, \alpha_i = \alpha^i$ means α is algebraic of degree n.
- In **2000** *Computing in Science and Engineering* named PSLQ one of the top 10 algorithms of the 20th century.

PROFILE: HELAMAN FERGUSON

Carving His Own Unique Niche, In Symbols and Stone

By refusing to choose between mathematics and art, a self-described "misfit" has found the place where parallel careers meet

CMS D.Borwein Prize

Madelung constant

CARMA

Experimental Mathematics: Integer Relation Methods

Secure Knowledge without Proof. Given real numbers $\beta, \alpha_1, \alpha_2, \ldots, \alpha_n$ Ferguson's integer relation method (PSLQ), finds a nontrivial linear relation of the form

$$a_0\beta + a_1\alpha_1 + a_2\alpha_2 + \cdots + a_n\alpha_n = 0, \qquad (1)$$

where a_i are integers — if one exists and provides an exclusion bound otherwise.

- If $a_0 \neq 0$ then (1) assures β is in rational vector space generated by $\{\alpha_1, \alpha_2, \ldots, \alpha_n\}$.
- $\beta = 1, \alpha_i = \alpha^i$ means α is algebraic of degree n.
- In **2000** *Computing in Science and Engineering* named PSLQ one of the top 10 algorithms of the 20th century.

PROFILE: HELAMAN FERGUSON

Carving His Own Unique Niche, In Symbols and Stone

By refusing to choose between mathematics and art, a self-described "misfit" has found the place where parallel careers meet

CMS D.Borwein Prize

Madelung constant

CARMA

Experimental Mathematics: Integer Relation Methods

Secure Knowledge without Proof. Given real numbers $\beta, \alpha_1, \alpha_2, \ldots, \alpha_n$ Ferguson's integer relation method (PSLQ), finds a nontrivial linear relation of the form

$$a_0\beta + a_1\alpha_1 + a_2\alpha_2 + \cdots + a_n\alpha_n = 0, \qquad (1)$$

where a_i are integers — if one exists and provides an exclusion bound otherwise.

- If $a_0 \neq 0$ then (1) assures β is in rational vector space generated by $\{\alpha_1, \alpha_2, \ldots, \alpha_n\}$.
- $\beta = 1, \alpha_i = \alpha^i$ means α is algebraic of degree n.
- In **2000** *Computing in Science and Engineering* named PSLQ one of the top 10 algorithms of the 20th century.

PROFILE: HELAMAN FERGUSON

Carving His Own Unique Niche, In Symbols and Stone

By refusing to choose between mathematics and art, a self-described "misfit" has found the place where parallel careers meet

CMS D.Borwein Prize

Madelung constant

Top Ten Algorithms

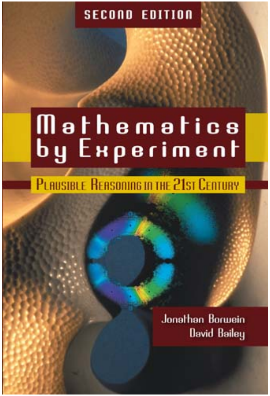
Algorithms for the Ages

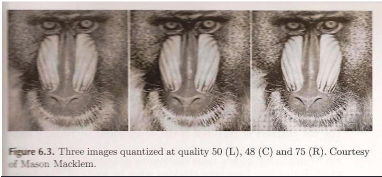
"Great algorithms are the poetry of computation," says Francis Sullivan of the Institute for Defense Analyses' Center for Computing Sciences in Bowie, Maryland. He and Jack Dongarra of the University of Tennessee and Oak Ridge National Laboratory have put together a sampling that might have made Robert Frost beam with pride--had the poet been a computer jock. Their list of 10 algorithms having "the greatest influence on the development and practice of science and engineering in the 20th century" appears in the January/February issue of Computing in Science & Engineering. If you use a computer, some of these algorithms are no doubt crunching your data as you read this. The drum roll, please:

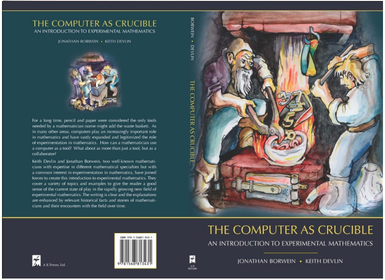
1. **1946: The Metropolis Algorithm for Monte Carlo.** Through the use of random processes, this algorithm offers an efficient way to stumble toward answers to problems that are too complicated to solve exactly.
2. **1947: Simplex Method for Linear Programming.** An elegant solution to a common problem in planning and decision-making.
3. **1950: Krylov Subspace Iteration Method.** A technique for rapidly solving the linear equations that abound in scientific computation.
4. **1951: The Decompositional Approach to Matrix Computations.** A suite of techniques for numerical linear algebra.
5. **1957: The Fortran Optimizing Compiler.** Turns high-level code into efficient computer-readable code.
6. **1959: QR Algorithm for Computing Eigenvalues.** Another crucial matrix operation made swift and practical.
7. **1962: Quicksort Algorithms for Sorting.** For the efficient handling of large databases.
8. **1965: Fast Fourier Transform.** Perhaps the most ubiquitous algorithm in use today, it breaks down waveforms (like sound) into periodic components.
9. **1977: Integer Relation Detection.** A fast method for spotting simple equations satisfied by collections of seemingly unrelated numbers.
10. **1987: Fast Multipole Method.** A breakthrough in dealing with the complexity of n-body calculations, applied in problems ranging from celestial mechanics to protein folding.

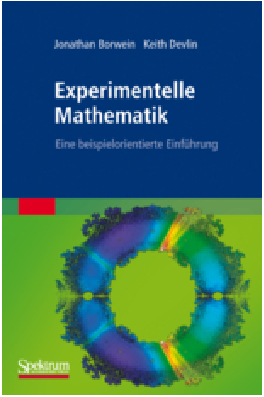
From *Random Samples*, Science page 799, February 4, 2000.

Experimental Mathematics: PSLQ is core to CARMA





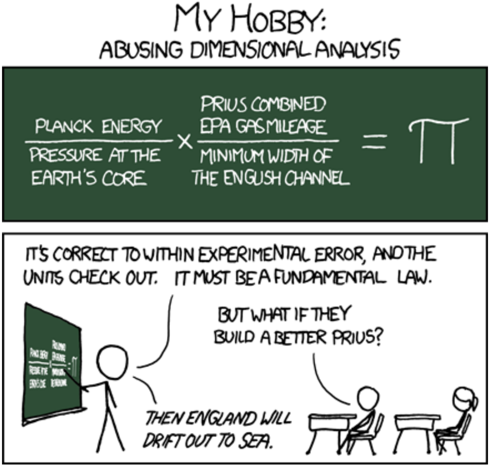




Experimental Mathematics (2004-08, 2009, 2010)

CARMA

Experimental Mathematics: KARMA takes many forms



... and there are always black swans

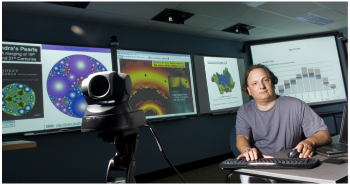
CARMA

CARMA's Mandate

toc

Mathematics, as "the language of high technology" which underpins all facets of modern life and current Information and Communication Technology (ICT), is ubiquitous. No other research centre exists focussing on the implications of developments in ICT, present and future, for the practice of research mathematics.

- CARMA fills this gap through exploitation and development of techniques and tools for computer-assisted discovery and disciplined data-mining including mathematical visualization.

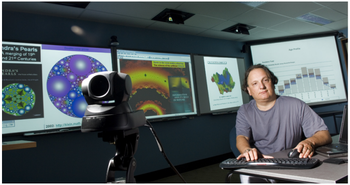


CARMA's Access Grid Room

CARMA's Mandate

Mathematics, as "the language of high technology" which underpins all facets of modern life and current Information and Communication Technology (ICT), is ubiquitous. No other research centre exists focussing on the implications of developments in ICT, present and future, for the practice of research mathematics.

- CARMA fills this gap through exploitation and development of techniques and tools for computer-assisted discovery and disciplined data-mining including mathematical visualization.



CARMA's Access Grid Room

CARMA's Objectives:

To perform R&D relating to the informed use of computers as an adjunct to mathematical discovery (including current advances in cognitive science, in information technology, operations research and theoretical computer science).

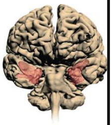
- of mathematics underlying computer-based decision support systems, particularly in automation and optimization of scheduling, planning and design activities, and to undertake mathematical modelling of such activities. (NUOR and partners)
- To promote and advise on the use of appropriate tools (hardware, software, databases, learning object repositories, mathematical knowledge management, collaborative technology) in academia, education and industry.
- To make the University of Newcastle a world-leading institution for Computer Assisted Research Mathematics and its Applications.[1]

CARMA

[1] 2010 ERA. UofN received the only '5' in Applied Mathematics

CARMA's Objectives:

To perform R&D relating to the informed use of computers as an adjunct to mathematical discovery (including current advances in cognitive science, in information technology, operations research and theoretical computer science).



- of mathematics underlying computer-based decision support systems, particularly in automation and optimization of scheduling, planning and design activities, and to undertake mathematical modelling of such activities. (NUOR and partners)
- To promote and advise on the use of appropriate tools (hardware, software, databases, learning object repositories, mathematical knowledge management, collaborative technology) in academia, education and industry.
- To make the University of Newcastle a world-leading institution for Computer Assisted Research Mathematics and its Applications.[1]

[1] 2010 ERA. UofN received the only '5' in Applied Mathematics

CARMA's Objectives:

To perform R&D relating to the informed use of computers as an adjunct to mathematical discovery (including current advances in cognitive science, in information technology, operations research and theoretical computer science).

- of mathematics underlying computer-based decision support systems, particularly in automation and optimization of scheduling, planning and design activities, and to undertake mathematical modelling of such activities. (NUOR and partners)
- To promote and advise on the use of appropriate tools (hardware, software, databases, learning object repositories, mathematical knowledge management, collaborative technology) in academia, education and industry.
- To make the University of Newcastle a world-leading institution for Computer Assisted Research Mathematics and its Applications.[1]

[1] 2010 ERA. UofN received the only '5' in Applied Mathematics

CARMA's Objectives:

To perform R&D relating to the informed use of computers as an adjunct to mathematical discovery (including current advances in cognitive science, in information technology, operations research and theoretical computer science).

- of mathematics underlying computer-based decision support systems, particularly in automation and optimization of scheduling, planning and design activities, and to undertake mathematical modelling of such activities. (NUOR and partners)
- To promote and advise on the use of appropriate tools (hardware, software, databases, learning object repositories, mathematical knowledge management, collaborative technology) in academia, education and industry.
- To make the University of Newcastle a world-leading institution for Computer Assisted Research Mathematics and its Applications.[1]

CARMA

[1] 2010 ERA. UofN received the only '5' in Applied Mathematics

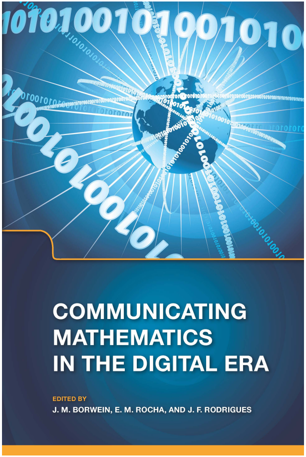
CARMA's Objectives:

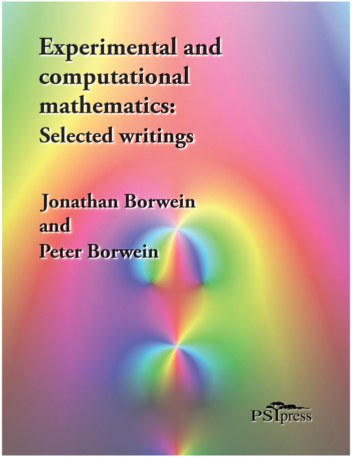
To perform R&D relating to the informed use of computers as an adjunct to mathematical discovery (including current advances in cognitive science, in information technology, operations research and theoretical computer science).

- of mathematics underlying computer-based decision support systems, particularly in automation and optimization of scheduling, planning and design activities, and to undertake mathematical modelling of such activities. (NUOR and partners)
- To promote and advise on the use of appropriate tools (hardware, software, databases, learning object repositories, mathematical knowledge management, collaborative technology) in academia, education and industry.
- To make the University of Newcastle a world-leading institution for Computer Assisted Research Mathematics and its Applications.[1]

[1] 2010 ERA. UofN received the only '5' in Applied Mathematics

Communication and Computation: are entangled



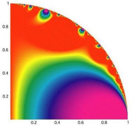


Communicating Mathematics (2008, 2010)

- See `http://carma.newcastle.edu.au/jon/c2c08.pdf` for chapter on Access Grid.

CARMA's Deep History

The co-evolution of symbolic/numeric (hybrid) computation, experimental mathematics, collaborative technology and **HPC**. (Experimentally found image took 3 hrs to print)



1982 PBB and JMB start 'minor' collaboration on fast computation at Dalhousie; becoming experimental mathematicians before the term was current.[2]

1993-03 Moved to SFU and founded Centre for Experimental and Constructive Mathematics (`www.cecm.sfu.ca`).

1995 Organic Mathematics Project: `www.cecm.sfu.ca/organics`

2004-09 JMB opens D-Drive (Dalhousie Distributed Research Institute and Virtual Environment) with Canada Research Chair funding

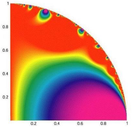
2004 PBB opens IRMACS (`www.irmacs.sfu.ca`) with CFI funds

2008 CARMA funded/ opened as Univ. Priority Research Centre

[2] *J. Experimental Mathematics* founded in 1993.

CARMA's Deep History

The co-evolution of symbolic/numeric (hybrid) computation, experimental mathematics, collaborative technology and **HPC**. (Experimentally found image took 3 hrs to print)



1982 PBB and JMB start 'minor' collaboration on fast computation at Dalhousie; becoming experimental mathematicians before the term was current.[2]

1993-03 Moved to SFU and founded Centre for Experimental and Constructive Mathematics (`www.cecm.sfu.ca`).

1995 Organic Mathematics Project: `www.cecm.sfu.ca/organics`

2004-09 JMB opens D-Drive (Dalhousie Distributed Research Institute and Virtual Environment) with Canada Research Chair funding

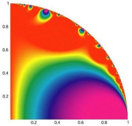
2004 PBB opens IRMACS (`www.irmacs.sfu.ca`) with CFI funds

2008 CARMA funded/ opened as Univ. Priority Research Centre

[2] *J. Experimental Mathematics* founded in 1993.

CARMA's Deep History

The co-evolution of symbolic/numeric (hybrid) computation, experimental mathematics, collaborative technology and **HPC**. (Experimentally found image took 3 hrs to print)



1982 PBB and JMB start 'minor' collaboration on fast computation at Dalhousie; becoming experimental mathematicians before the term was current.[2]

1993-03 Moved to SFU and founded Centre for Experimental and Constructive Mathematics (`www.cecm.sfu.ca`).

1995 Organic Mathematics Project: `www.cecm.sfu.ca/organics`

2004-09 JMB opens D-Drive (Dalhousie Distributed Research Institute and Virtual Environment) with Canada Research Chair funding

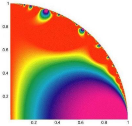
2004 PBB opens IRMACS (`www.irmacs.sfu.ca`) with CFI funds

2008 CARMA funded/ opened as Univ. Priority Research Centre

[2] *J. Experimental Mathematics* founded in 1993.

CARMA's Deep History

The co-evolution of symbolic/numeric (hybrid) computation, experimental mathematics, collaborative technology and **HPC**. (Experimentally found image took 3 hrs to print)



1982 PBB and JMB start 'minor' collaboration on fast computation at Dalhousie; becoming experimental mathematicians before the term was current.[2]

1993-03 Moved to SFU and founded Centre for Experimental and Constructive Mathematics (`www.cecm.sfu.ca`).

1995 Organic Mathematics Project: `www.cecm.sfu.ca/organics`

2004-09 JMB opens D-Drive (Dalhousie Distributed Research Institute and Virtual Environment) with Canada Research Chair funding

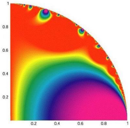
2004 PBB opens IRMACS (`www.irmacs.sfu.ca`) with CFI funds

2008 CARMA funded/ opened as Univ. Priority Research Centre

[2] *J. Experimental Mathematics* founded in 1993.

CARMA's Deep History

The co-evolution of symbolic/numeric (hybrid) computation, experimental mathematics, collaborative technology and **HPC**. (Experimentally found image took 3 hrs to print)



1982 PBB and JMB start 'minor' collaboration on fast computation at Dalhousie; becoming experimental mathematicians before the term was current.[2]

1993-03 Moved to SFU and founded Centre for Experimental and Constructive Mathematics (`www.cecm.sfu.ca`).

1995 Organic Mathematics Project: `www.cecm.sfu.ca/organics`

2004-09 JMB opens D-Drive (Dalhousie Distributed Research Institute and Virtual Environment) with Canada Research Chair funding

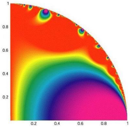
2004 PBB opens IRMACS (`www.irmacs.sfu.ca`) with CFI funds

2008 CARMA funded/ opened as Univ. Priority Research Centre

[2] *J. Experimental Mathematics* founded in 1993.

CARMA's Deep History

The co-evolution of symbolic/numeric (hybrid) computation, experimental mathematics, collaborative technology and **HPC**. (Experimentally found image took 3 hrs to print)



1982 PBB and JMB start 'minor' collaboration on fast computation at Dalhousie; becoming experimental mathematicians before the term was current.[2]

1993-03 Moved to SFU and founded Centre for Experimental and Constructive Mathematics (`www.cecm.sfu.ca`).

1995 Organic Mathematics Project: `www.cecm.sfu.ca/organics`

2004-09 JMB opens D-Drive (Dalhousie Distributed Research Institute and Virtual Environment) with Canada Research Chair funding

2004 PBB opens IRMACS (`www.irmacs.sfu.ca`) with CFI funds

2008 CARMA funded/ opened as Univ. Priority Research Centre

[2] *J. Experimental Mathematics* founded in 1993.

CARMA's Structure

toc

Roughly **30** current Members and Associates:

- Steering Committee (Assoc Directors for Applied/Pure/Stats)
- External Advisory Committee (IBM, Melbourne, LBNL)
- Members and Students from Newcastle
- Associate Members from Everywhere
- Scientific and Administrative Officers

Frequent visitors: both student and faculty, short and long-term

CARMA's AMSI AGR and **Inner Sanctum Rooms**

CARMA

CARMA's Structure

Roughly **30** current Members and Associates:

- Steering Committee (Assoc Directors for Applied/Pure/Stats)
- External Advisory Committee (IBM, Melbourne, LBNL)
- Members and Students from Newcastle
- Associate Members from Everywhere
- Scientific and Administrative Officers

Frequent visitors: both student and faculty, short and long-term

CARMA's AMSI AGR and **Inner Sanctum Rooms**

CARMA's Structure

Roughly **30** current Members and Associates:

- Steering Committee (Assoc Directors for Applied/Pure/Stats)
- External Advisory Committee (IBM, Melbourne, LBNL)
- Members and Students from Newcastle
- Associate Members from Everywhere
- Scientific and Administrative Officers

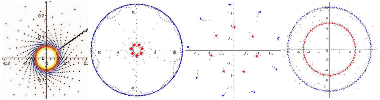
Frequent visitors: both student and faculty, short and long-term

CARMA's AMSI AGR and **Inner Sanctum Rooms**

Continuing Scientific Activities Include

toc

- **Regular Colloquia and Seminar Series**
  - NUOR, SigmaOpt, Discrete Maths, Analysis and Number Theory



- **AMSI Access Grid Activities**: www.amsi.org.au
  - ANZIAM SIGMAopt Seminar with UoSA and RMIT http://sigmaopt.newcastle.edu.au
  - Trans Pacific Workshop: with UBC-O and SFU (monthly-ish)
  - Short Lecture Series (2-5 lectures)

    2010 Rockafellar on *Risk* and Diestel on *Haar measure*

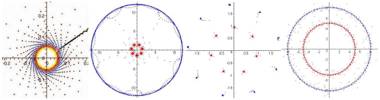
    2011 Cominetti on *Scheduling* and Zhu on *Finance*
  - AMSI Honours (MSc) Courses (400 hours per term)

- **International Workshops and Conferences**:
  - IP Down Under Satellite for INFORS 2011 (July 6-8, 2011)
  - Number Th. in Honour of Alf Van der Poorten (March, 2012)
  - ANZIAM 2013 (Jan 27-31, 2013)

Continuing Scientific Activities Include

toc

- **Regular Colloquia and Seminar Series**
  - NUOR, SigmaOpt, Discrete Maths, Analysis and Number Theory



- **AMSI Access Grid Activities**: www.amsi.org.au
  - ANZIAM SIGMAopt Seminar with UoSA and RMIT http://sigmaopt.newcastle.edu.au
  - Trans Pacific Workshop: with UBC-O and SFU (monthly-ish)
  - Short Lecture Series (2-5 lectures)

    2010 Rockafellar on *Risk* and Diestel on *Haar measure*

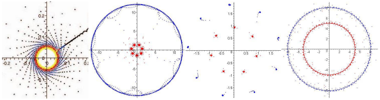
    2011 Cominetti on *Scheduling* and Zhu on *Finance*
  - AMSI Honours (MSc) Courses (400 hours per term)
- **International Workshops and Conferences**:
  - IP Down Under Satellite for INFORS 2011 (July 6-8, 2011)
  - Number Th. in Honour of Alf Van der Poorten (March, 2012)
  - ANZIAM 2013 (Jan 27-31, 2013)

CARMA

Continuing Scientific Activities Include

toc

- **Regular Colloquia and Seminar Series**
  - NUOR, SigmaOpt, Discrete Maths, Analysis and Number Theory



- **AMSI Access Grid Activities**: www.amsi.org.au
  - ANZIAM SIGMAopt Seminar with UoSA and RMIT http://sigmaopt.newcastle.edu.au
  - Trans Pacific Workshop: with UBC-O and SFU (monthly-ish)
  - Short Lecture Series (2-5 lectures)

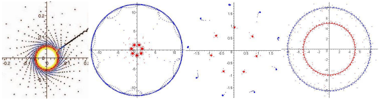
    2010 Rockafellar on *Risk* and Diestel on *Haar measure*

    2011 Cominetti on *Scheduling* and Zhu on *Finance*
  - AMSI Honours (MSc) Courses (400 hours per term)
- **International Workshops and Conferences**:
  - IP Down Under Satellite for INFORS 2011 (July 6-8, 2011)
  - Number Th. in Honour of Alf Van der Poorten (March, 2012)
  - ANZIAM 2013 (Jan 27-31, 2013)

Continuing Scientific Activities Include

toc

- **Regular Colloquia and Seminar Series**
  - NUOR, SigmaOpt, Discrete Maths, Analysis and Number Theory



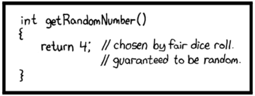
- **AMSI Access Grid Activities**: www.amsi.org.au
  - ANZIAM SIGMAopt Seminar with UoSA and RMIT
    http://sigmaopt.newcastle.edu.au
  - Trans Pacific Workshop: with UBC-O and SFU (monthly-ish)
  - Short Lecture Series (2-5 lectures)
    - **2010** Rockafellar on *Risk* and Diestel on *Haar measure*
    - **2011** Cominetti on *Scheduling* and Zhu on *Finance*
  - AMSI Honours (MSc) Courses (400 hours per term)

- **International Workshops and Conferences**:
  - IP Down Under Satellite for INFORS 2011 (July 6-8, 2011)
  - Number Th. in Honour of Alf Van der Poorten (March, 2012)
  - ANZIAM 2013 (Jan 27-31, 2013)

Our Services Include

toc

AGR Grid-enabled interconnected rooms for classes, seminars, meetings: Likely to become HQ for AMSI AGRs + NeCTAR?



V205 for dis-located collaboration;

V206 for co-located collaboration.

HPC 64 core MacPro Cluster and x-grid plus access to NSW and National computing services.

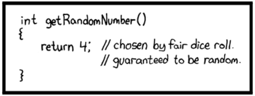
Web **Services** include:

- DocServer http://docserver.carma.newcastle.edu.au: CECM → DDRIVE → CARMA Archie → Mosaic → Google
- Inverse symbolic calculator (ISC Plus) http://isc.carma.newcastle.edu.au
- BBP digit database http://bbp.carma.newcastle.edu.au
- The Top Ten Numbers University Outreach http://numbers.carma.newcastle.edu.au
- Ask CARMA http://ask.carma.newcastle.edu.au for School Outreach: β-test

Our Services Include

toc

AGR Grid-enabled interconnected rooms for classes, seminars, meetings: Likely to become HQ for AMSI AGRs + NeCTAR?



V205 for dis-located collaboration;

V206 for co-located collaboration.

HPC 64 core MacPro Cluster and x-grid plus access to NSW and National computing services.

Web **Services** include:

- DocServer http://docserver.carma.newcastle.edu.au: CECM → DDRIVE → CARMA Archie → Mosaic → Google
- Inverse symbolic calculator (ISC Plus) http://isc.carma.newcastle.edu.au
- BBP digit database http://bbp.carma.newcastle.edu.au
- The Top Ten Numbers University Outreach http://numbers.carma.newcastle.edu.au
- Ask CARMA http://ask.carma.newcastle.edu.au for School Outreach: β-test

CARMA

Our Services Include

toc

AGR Grid-enabled interconnected rooms for classes, seminars, meetings: Likely to become HQ for AMSI AGRs + NeCTAR?

```
int getRandomNumber()
{
    return 4;  // chosen by fair dice roll.
               // guaranteed to be random.
}
```

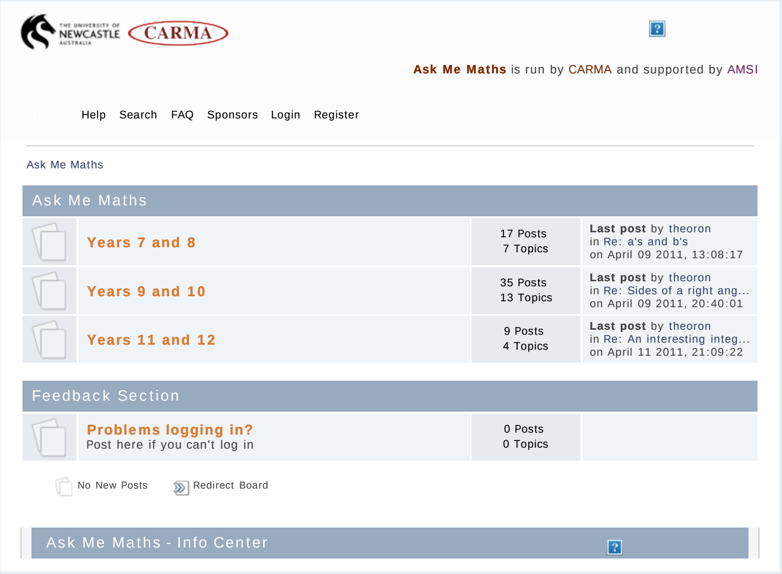
V205 for dis-located collaboration;

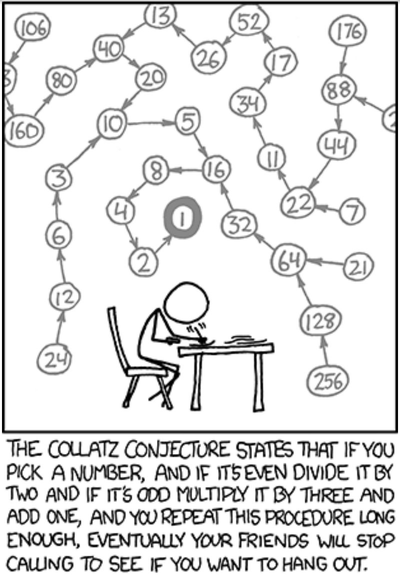
V206 for co-located collaboration.

HPC 64 core MacPro Cluster and x-grid plus access to NSW and National computing services.

Web **Services** include:

- DocServer `http://docserver.carma.newcastle.edu.au`:
  CECM → DDRIVE → CARMA   Archie → Mosaic → Google
- Inverse symbolic calculator (ISC Plus)
  `http://isc.carma.newcastle.edu.au`
- BBP digit database `http://bbp.carma.newcastle.edu.au`
- The Top Ten Numbers University Outreach
  `http://numbers.carma.newcastle.edu.au`
- Ask CARMA `http://ask.carma.newcastle.edu.au` for School Outreach: β-test


THE UNIVERSITY OF NEWCASTLE AUSTRALIA
CARMA
Ask Me Maths is run by CARMA and supported by AMSI
Help Search FAQ Sponsors Login Register
Ask Me Maths
Ask Me Maths
Years 7 and 8
17 Posts
7 Topics
Last post by theoron
in Re: a's and b's
on April 09 2011, 13:08:17
Years 9 and 10
35 Posts
13 Topics
Last post by theoron
in Re: Sides of a right ang...
on April 09 2011, 20:40:01
Years 11 and 12
9 Posts
4 Topics
Last post by theoron
in Re: An interesting integ...
on April 11 2011, 21:09:22
Feedback Section
Problems logging in?
Post here if you can't log in
0 Posts
0 Topics
No New Posts
Redirect Board
Ask Me Maths - Info Center



Math Drudge: `http://experimentalmath.info/blog/2011/06/has-the-3n1-conjecture-been-proved/`

Other

Compute Canada's **Engines of Discovery:** Executive **Summary** (ENIAC and Story)

The one true Larry

Pi (Life of Pi (2010))

Recent or Notable Items:

2012

March 12-16, 2012. Number Theory Conference in Memory of Alf van der Poorten at CARMA.

2011

June 16, 2011. Second semester AMSI Honours Course on MZVs (Borwein-Zudilin).
June 2-4. JMB at the World Science Festival
May 16-20. JonFest at the IRMACS centre. (Pictures and videos of lectures available.)
April 19. Blue Gene BBP article from Australian. Also Pi, HPCnet and energy.gov.
April 5. Interactive BBP digit database online.
March 15. My AGR PiDay Talk V206 at Univ of Newcastle at 10am (Details)
March 14. My webcast PiDay Talk from University of Technology Sydney
Details, RECORD Blue-Gene Computations and Press Release
March 10. Happy Pi Day: The infinite appeal of Pi.
Feb 1. Newcastle Applied Maths ranked top in Australia.

2010

ResearcherID Profile Researcherid.com

Dr. Jonathan M. Borwein
FRSC FAAAS FBAS FAA

Previous Webpage

BIO

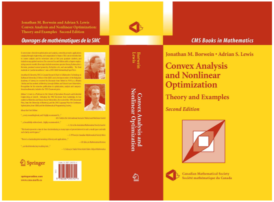
(Pics)

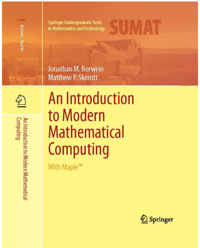
Book Covers

Current Research Interests Include

toc

1. Optimization Theory and Applications
   - Inverse problems & Phase reconstruction
   - Projection methods & Entropy optimization
   - Signal & (Medical) Image reconstruction
2. Nonlinear Functional Analysis
   - Convex analysis and Monotone operators
   - Geometric fixed point theory
3. Computational Number Theory
   - Arithmetic of random walks
   - Mahler measures of polynomials
   - Algorithms for Special Functions
   - Pi & friends — and JB-AvdP-WZ book.
4. Algorithmic Complexity Theory
   - Fast extreme precision computation
   - Multidimensional numerical quadrature
   - Mathematical visualization (and 3D)



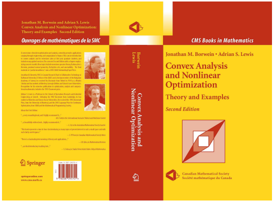


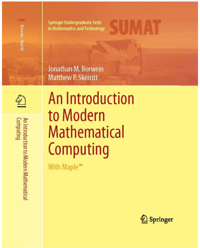
CARMA

Current Research Interests Include

toc

1. Optimization Theory and Applications
   - Inverse problems & Phase reconstruction
   - Projection methods & Entropy optimization
   - Signal & (Medical) Image reconstruction
2. Nonlinear Functional Analysis
   - Convex analysis and Monotone operators
   - Geometric fixed point theory
3. Computational Number Theory
   - Arithmetic of random walks
   - Mahler measures of polynomials
   - Algorithms for Special Functions
   - Pi & friends — and JB-AvdP-WZ book.
4. Algorithmic Complexity Theory
   - Fast extreme precision computation
   - Multidimensional numerical quadrature
   - Mathematical visualization (and 3D)



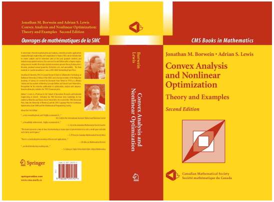


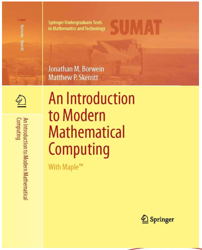
CARMA

Current Research Interests Include

toc

1. Optimization Theory and Applications
   - Inverse problems & Phase reconstruction
   - Projection methods & Entropy optimization
   - Signal & (Medical) Image reconstruction
2. Nonlinear Functional Analysis
   - Convex analysis and Monotone operators
   - Geometric fixed point theory
3. Computational Number Theory
   - Arithmetic of random walks
   - Mahler measures of polynomials
   - Algorithms for Special Functions
   - Pi & friends — and JB-AvdP-WZ book.
4. Algorithmic Complexity Theory
   - Fast extreme precision computation
   - Multidimensional numerical quadrature
   - Mathematical visualization (and 3D)



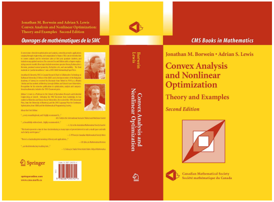


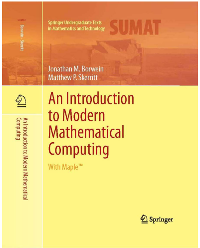
CARMA

Current Research Interests Include

toc

1. Optimization Theory and Applications
   - Inverse problems & Phase reconstruction
   - Projection methods & Entropy optimization
   - Signal & (Medical) Image reconstruction
2. Nonlinear Functional Analysis
   - Convex analysis and Monotone operators
   - Geometric fixed point theory
3. Computational Number Theory
   - Arithmetic of random walks
   - Mahler measures of polynomials
   - Algorithms for Special Functions
   - Pi & friends — and JB-AvdP-WZ book.
4. Algorithmic Complexity Theory
   - Fast extreme precision computation
   - Multidimensional numerical quadrature
   - Mathematical visualization (and 3D)



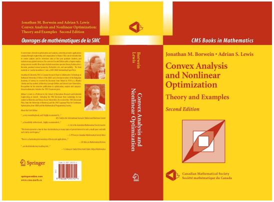


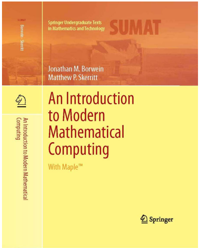
CARMA

Current Research Interests Include

toc

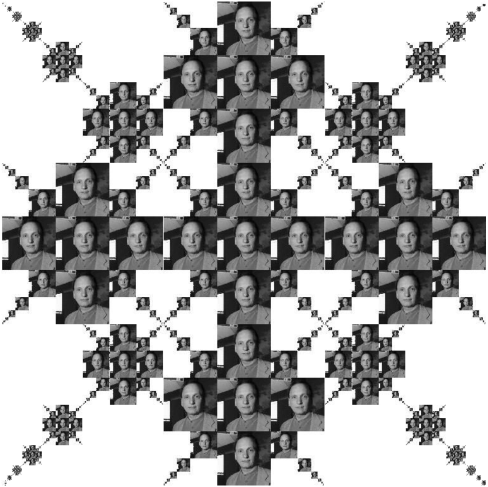
1. Optimization Theory and Applications
   - Inverse problems & Phase reconstruction
   - Projection methods & Entropy optimization
   - Signal & (Medical) Image reconstruction
2. Nonlinear Functional Analysis
   - Convex analysis and Monotone operators
   - Geometric fixed point theory
3. Computational Number Theory
   - Arithmetic of random walks
   - Mahler measures of polynomials
   - Algorithms for Special Functions
   - Pi & friends — and JB-AvdP-WZ book.
4. Algorithmic Complexity Theory
   - Fast extreme precision computation
   - Multidimensional numerical quadrature
   - Mathematical visualization (and 3D)



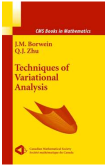


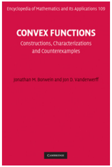
CARMA

The Fractal Nature of Me: Examples of Each



1. Divide and Concur: Douglas-Rachford methods for phase reconstruction
2. Three Optimization Texts — one on previous page:

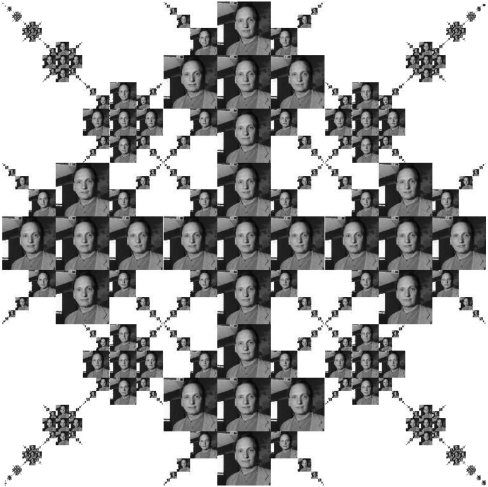




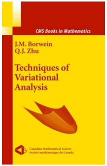
3. Short Random Walks
4. Single Digit Algorithms: BBP for π, π^2, G.

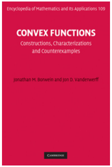
CARMA

The Fractal Nature of Me: Examples of Each



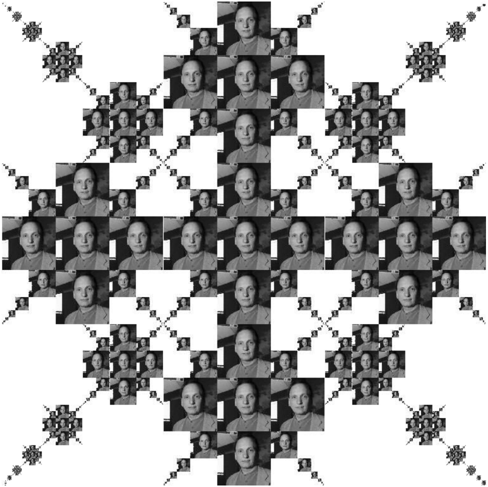
1. Divide and Concur: Douglas-Rachford methods for phase reconstruction
2. Three Optimization Texts — one on previous page:



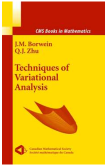


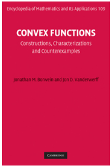
3. Short Random Walks
4. Single Digit Algorithms: BBP for π, π^2, G.

The Fractal Nature of Me: Examples of Each



1. Divide and Concur: Douglas-Rachford methods for phase reconstruction
2. Three Optimization Texts — one on previous page:

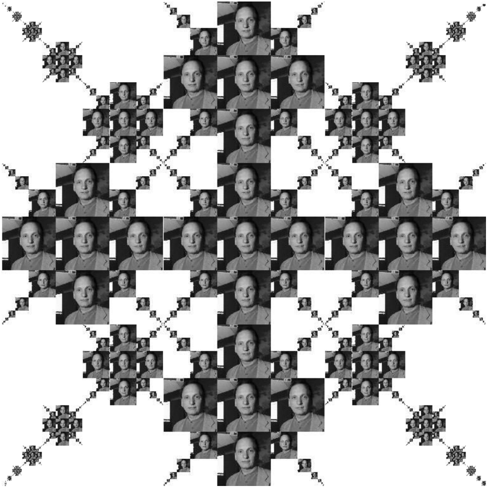




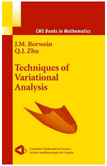
3. Short Random Walks
4. Single Digit Algorithms: BBP for π, π^2, G.

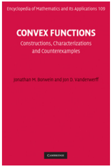
CARMA

The Fractal Nature of Me: Examples of Each



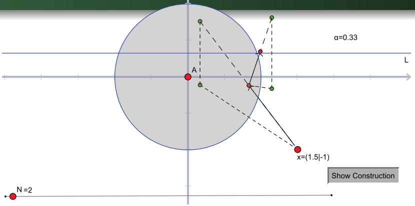
1. Divide and Concur: Douglas-Rachford methods for phase reconstruction
2. Three Optimization Texts — one on previous page:

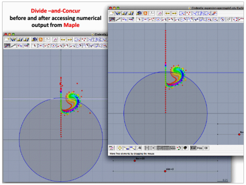


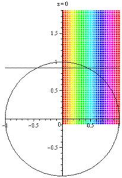


3. Short Random Walks
4. Single Digit Algorithms: BBP for π, π^2, G.

1. . . . Visual Theorems: Reflect-Reflect-Average





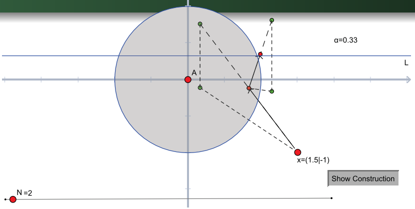


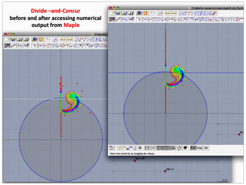
To find a point on a sphere and in an affine subspace

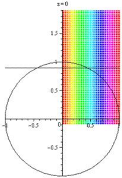
Briefly, a visual theorem is the graphical or visual output from a computer program — usually one of a family of such outputs — which the eye organizes into a coherent, identifiable whole and which is able to inspire mathematical questions of a traditional nature or which contributes in some way to our understanding or enrichment of some mathematical or real world situation.

— Davis, 1993, p. 333.

1. . . . Visual Theorems: Reflect-Reflect-Average







To find a point on a sphere and in an affine subspace

> *Briefly, a visual theorem is the graphical or visual output from a computer program — usually one of a family of such outputs — which the eye organizes into a coherent, identifiable whole and which is able to inspire mathematical questions of a traditional nature or which contributes in some way to our understanding or enrichment of some mathematical or real world situation.*
> — *Davis, 1993, p. 333.*

3. Three Ramblers: Straub, Borwein, Wan

3. Moments of Random Walks (Flights)

Definition (Moments)

For complex s the n-th moment function is

$$W_n(s) = \int_{[0,1]^n} \left| \sum_{k=1}^{n} e^{2\pi x_k i} \right|^s \mathrm{d}\boldsymbol{x}$$

$$= \int_{[0,1]^{n-1}} \left| 1 + \sum_{k=1}^{n-1} e^{2\pi x_k i} \right|^s \mathrm{d}(x_1, \ldots, x_{n-1})$$

Thus, $W_n := W_n(1)$ is the *expectation*.

- So

$$W_2 = 4 \int_0^{1/4} \cos(\pi x)\,\mathrm{d}x = \frac{4}{\pi}$$

and $W_2(s) = \binom{s/2}{s}$ (combinatorics).

CARMA

3. Moments of Random Walks (Flights)

Definition (Moments)

For complex s the n-th moment function is

$$W_n(s) = \int_{[0,1]^n} \left| \sum_{k=1}^{n} e^{2\pi x_k i} \right|^s \mathrm{d}\boldsymbol{x}$$

$$= \int_{[0,1]^{n-1}} \left| 1 + \sum_{k=1}^{n-1} e^{2\pi x_k i} \right|^s \mathrm{d}(x_1, \ldots, x_{n-1})$$

Thus, $W_n := W_n(1)$ is the *expectation*.

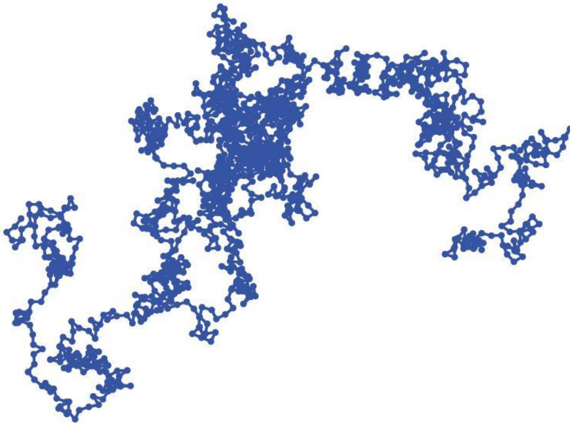
- So

$$W_2 = 4 \int_0^{1/4} \cos(\pi x)\, \mathrm{d}x = \frac{4}{\pi}$$

and $W_2(s) = \binom{s/2}{s}$ (combinatorics).

CARMA

3. One 1500-step Walk in the plane: a familiar picture

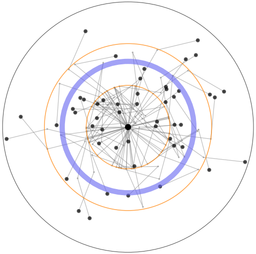


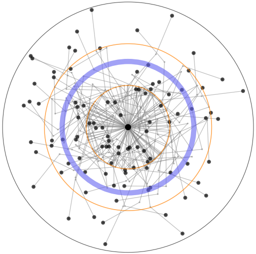
2D and 3D lattice walks are different:

> *A drunk man will find his way home but a drunk bird may get lost forever.*
> — *Shizuo Kakutani*

3. 50, 100, 1000 3-step Walks: a less familiar picture?

$$W_3(1) = \frac{16\sqrt[3]{4}\,\pi^2}{\Gamma(\frac{1}{3})^6} + \frac{3\Gamma(\frac{1}{3})^6}{8\sqrt[3]{4}\,\pi^4}$$





3. Moments of a Three Step Walk: in the complex plane

Theorem (Tractable hypergeometric form for W_3)

(a) *For* $s \neq -3, -5, -7, \ldots$, *we have*

$$W_3(s) = \frac{3^{s+3/2}}{2\pi}\,\beta\left(s+\frac{1}{2}, s+\frac{1}{2}\right)\,{}_3F_2\left(\left.{\frac{s+2}{2}, \frac{s+2}{2}, \frac{s+2}{2} \atop 1, \frac{s+3}{2}}\right|\frac{1}{4}\right). \tag{2}$$

(b) *For every natural number* $k = 1, 2, \ldots$,

$$W_3(-2k-1) = \frac{\sqrt{3}\,\binom{2k}{k}^2}{2^{4k+1}3^{2k}}\,{}_3F_2\left(\left.{\frac{1}{2}, \frac{1}{2}, \frac{1}{2} \atop k+1, k+1}\right|\frac{1}{4}\right).$$

CARMA

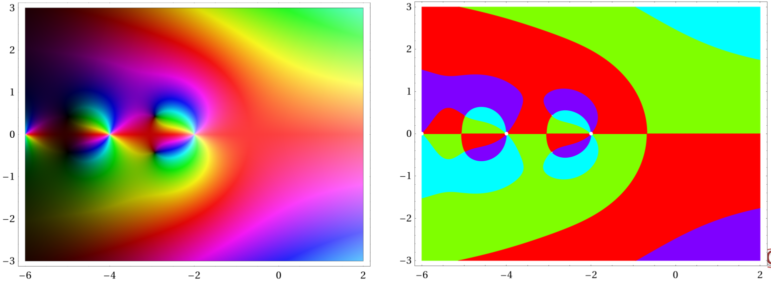
3. Moments of a Four Step Walk

Theorem (Meijer-G form for W_4)

For $\operatorname{Re} s > -2$ *and* s *not an odd integer*

$$W_4(s) = \frac{2^s}{\pi} \frac{\Gamma(1+\frac{s}{2})}{\Gamma(-\frac{s}{2})} G^{22}_{44}\left(\begin{matrix} 1, \frac{1-s}{2}, 1, 1 \\ \frac{1}{2} - \frac{s}{2}, -\frac{s}{2}, -\frac{s}{2}, -\frac{s}{2} \end{matrix} \middle| 1 \right). \tag{3}$$

W_4 with phase colored continuously (L) and by quadrant (R)



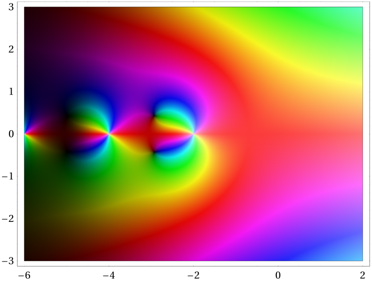
3. Moments of a Four Step Walk

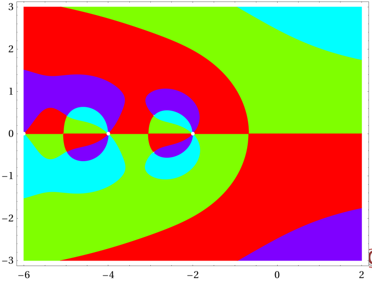
Theorem (Meijer-G form for W_4)

For $\mathrm{Re}\ s > -2$ *and* s *not an odd integer*

$$W_4(s) = \frac{2^s}{\pi} \frac{\Gamma(1+\frac{s}{2})}{\Gamma(-\frac{s}{2})} G^{22}_{44}\left(\begin{matrix} 1, \frac{1-s}{2}, 1, 1 \\ \frac{1}{2} - \frac{s}{2}, -\frac{s}{2}, -\frac{s}{2}, -\frac{s}{2} \end{matrix} \middle| 1 \right). \tag{3}$$

W_4 with phase colored continuously (L) and by quadrant (R)

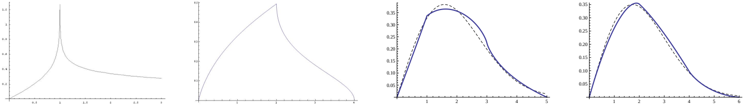




CARMA

3. Density of a Three and Four Step Walk (BSW, 2010)

$$p_3(\alpha) = \frac{2\sqrt{3}\alpha}{\pi\,(3+\alpha^2)}\,{}_2F_1\left(\left.{\frac{1}{3},\frac{2}{3} \atop 1}\right|\frac{\alpha^2\left(9-\alpha^2\right)^2}{\left(3+\alpha^2\right)^3}\right)$$

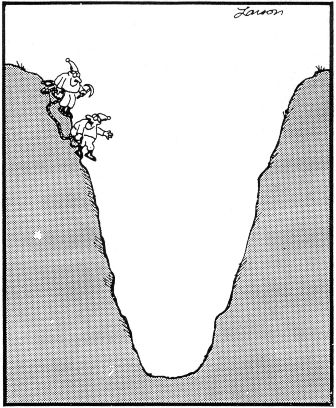


For $n \geq 7$ the asymptotics $p_n(x) \sim \frac{2x}{n} e^{-x^2/n}$ are good.
(These are hard to draw.)

$$p_4(\alpha) = \frac{2}{\pi^2}\frac{\sqrt{16-\alpha^2}}{\alpha}\,\mathrm{Re}\;{}_3F_2\left(\left.{\frac{1}{2},\frac{1}{2},\frac{1}{2} \atop \frac{5}{6},\frac{7}{6}}\right|\frac{\left(16-\alpha^2\right)^3}{108\,\alpha^4}\right).$$

4. BBP Digits Extraction Algorithms

- *Notices AMS* in press:
  carma.newcastle.edu.au/jon/bbp-bluegene.pdf

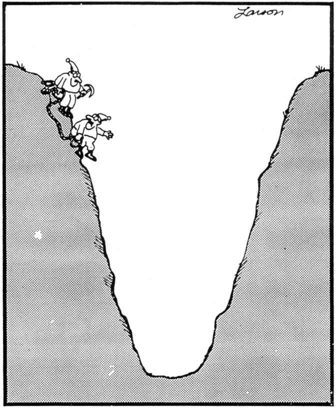


"Because it's not there."

CARMA

4. BBP Digits Extraction Algorithms

- *Notices AMS* in press:
  `carma.newcastle.edu.au/jon/bbp-bluegene.pdf`



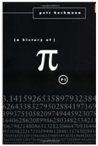
"Because it's not there."

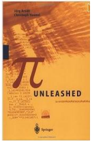
Computing Individual Digits of π

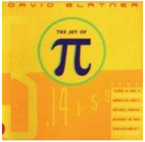
TOC IBM

1971. One might think everything of interest about computing π has been discovered. This was Beckmann's view in *A History of* π

Yet, the Salamin-Brent quadratic iteration was found only five years later. Higher-order algorithms followed in the 1980s.







1990. Rabinowitz and Wagon found a 'spigot' algorithm for π: It 'drips' individual digits (of π in any desired base) using all previous digits.

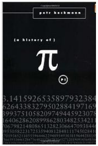
But even insiders are sometimes surprised by a new discovery: in this case **BBP series**.

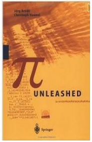
CARMA

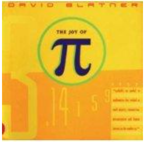
Computing Individual Digits of π

1971. One might think everything of interest about computing π has been discovered. This was Beckmann's view in *A History of π*

Yet, the Salamin-Brent quadratic iteration was found only five years later. Higher-order algorithms followed in the 1980s.







1990. Rabinowitz and Wagon found a 'spigot' algorithm for π: It 'drips' individual digits (of π in any desired base) using all previous digits.

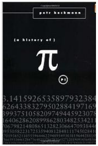
But even insiders are sometimes surprised by a new discovery: in this case **BBP series**.

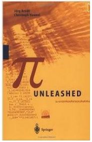
CARMA

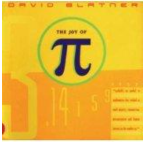
Computing Individual Digits of π

1971. One might think everything of interest about computing π has been discovered. This was Beckmann's view in *A History of π*

Yet, the Salamin-Brent quadratic iteration was found only five years later. Higher-order algorithms followed in the 1980s.







1990. Rabinowitz and Wagon found a 'spigot' algorithm for π: It 'drips' individual digits (of π in any desired base) using all previous digits.

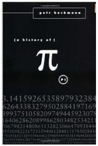
But even insiders are sometimes surprised by a new discovery: in this case BBP series.

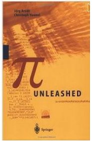
Computing Individual Digits of π

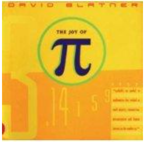
TOC IBM

1971. One might think everything of interest about computing π has been discovered. This was Beckmann's view in *A History of π*

Yet, the Salamin-Brent quadratic iteration was found only five years later. Higher-order algorithms followed in the 1980s.







1990. Rabinowitz and Wagon found a 'spigot' algorithm for π: It 'drips' individual digits (of π in any desired base) using all previous digits.

But even insiders are sometimes surprised by a new discovery: in this case **BBP series**.

CARMA

What a BBP Algorithm Does?

Prior to **1996**, most folks thought to compute the d-th digit of π, you had to generate the (order of) the entire first d digits.

- **This is not true**, at least for hex (base 16) or binary (base 2) digits of π. In **1996**, P. Borwein, Plouffe, and Bailey found an algorithm for individual hex digits of π. A BBP algorithm is one that produces:

- a modest-length string hex or binary digits of π (or other constants) beginning at an any position, *using no prior bits*;
  1. is implementable on any modern computer;
  2. requires no multiple precision software;
  3. requires very little memory; and has
  4. a computational cost growing only slightly faster than the digit position.

CARMA

What a BBP Algorithm Does?

Prior to **1996**, most folks thought to compute the d-th digit of π, you had to generate the (order of) the entire first d digits.

- **This is not true**, at least for hex (base 16) or binary (base 2) digits of π. In **1996**, P. Borwein, Plouffe, and Bailey found an algorithm for individual hex digits of π. A BBP algorithm is one that produces:

- a modest-length string hex or binary digits of π (or other constants) beginning at an any position, *using no prior bits*;
  1. is implementable on any modern computer;
  2. requires no multiple precision software;
  3. requires very little memory; and has
  4. a computational cost growing only slightly faster than the digit position.

CARMA

What a BBP Algorithm Does?

Prior to **1996**, most folks thought to compute the d-th digit of π, you had to generate the (order of) the entire first d digits.

- **This is not true**, at least for hex (base 16) or binary (base 2) digits of π. In **1996**, P. Borwein, Plouffe, and Bailey found an algorithm for individual hex digits of π. A BBP algorithm is one that produces:

- a modest-length string hex or binary digits of π (or other constants) beginning at an any position, *using no prior bits*;
  1. is implementable on any modern computer;
  2. requires no multiple precision software;
  3. requires very little memory; and has
  4. a computational cost growing only slightly faster than the digit position.

What a BBP Algorithm Does?

Prior to **1996**, most folks thought to compute the d-th digit of π, you had to generate the (order of) the entire first d digits.

- **This is not true**, at least for hex (base 16) or binary (base 2) digits of π. In **1996**, P. Borwein, Plouffe, and Bailey found an algorithm for individual hex digits of π. A BBP algorithm is one that produces:

- a modest-length string hex or binary digits of π (or other constants) beginning at an any position, *using no prior bits*;
  1. is implementable on any modern computer;
  2. requires no multiple precision software;
  3. requires very little memory; and has
  4. a computational cost growing only slightly faster than the digit position.

CARMA

What a BBP Algorithm Does?

Prior to **1996**, most folks thought to compute the d-th digit of π, you had to generate the (order of) the entire first d digits.

- **This is not true**, at least for hex (base 16) or binary (base 2) digits of π. In **1996**, P. Borwein, Plouffe, and Bailey found an algorithm for individual hex digits of π. A BBP algorithm is one that produces:

- a modest-length string hex or binary digits of π (or other constants) beginning at an any position, *using no prior bits*;
  1. is implementable on any modern computer;
  2. requires no multiple precision software;
  3. requires very little memory; and has
  4. a computational cost growing only slightly faster than the digit position.

CARMA

What BBP Is? Reverse Engineered Mathematics

This is based on the following then new formula for π:

$$\pi = \sum_{i=0}^{\infty} \frac{1}{16^i} \left(\frac{4}{8i+1} - \frac{2}{8i+4} - \frac{1}{8i+5} - \frac{1}{8i+6} \right) \tag{4}$$

- The millionth hex digit (four millionth binary digit) of π can be found in under **30** secs on a fairly new computer in Maple (not C++) and the billionth in **10** hrs.

Equation (4) was discovered numerically using integer relation methods over months in our Vancouver lab, **CECM**. It arrived in the coded form:

$$\pi = 4\,{}_2F_1\left(1, \frac{1}{4}; \frac{5}{4}, -\frac{1}{4}\right) + 2\tan^{-1}\left(\frac{1}{2}\right) - \log 5$$

where ${}_2F_1(1, 1/4; 5/4, -1/4) = 0.955933837\ldots$ is a Gauss hypergeometric function.

CARMA

What BBP Is? Reverse Engineered Mathematics

This is based on the following then new formula for π:

$$\pi = \sum_{i=0}^{\infty} \frac{1}{16^i} \left(\frac{4}{8i+1} - \frac{2}{8i+4} - \frac{1}{8i+5} - \frac{1}{8i+6} \right) \tag{4}$$

- The millionth hex digit (four millionth binary digit) of π can be found in under **30** secs on a fairly new computer in Maple (not C++) and the billionth in **10** hrs.

Equation (4) was discovered numerically using integer relation methods over months in our Vancouver lab, **CECM**. It arrived in the coded form:

$$\pi = 4\,{}_2F_1\left(1, \frac{1}{4}; \frac{5}{4}, -\frac{1}{4}\right) + 2\tan^{-1}\left(\frac{1}{2}\right) - \log 5$$

where ${}_2F_1(1, 1/4; 5/4, -1/4) = 0.955933837\ldots$ is a Gauss hypergeometric function.

What BBP Is? Reverse Engineered Mathematics

This is based on the following then new formula for π:

$$\pi = \sum_{i=0}^{\infty} \frac{1}{16^i} \left(\frac{4}{8i+1} - \frac{2}{8i+4} - \frac{1}{8i+5} - \frac{1}{8i+6} \right) \tag{4}$$

- The millionth hex digit (four millionth binary digit) of π can be found in under **30** secs on a fairly new computer in Maple (not C++) and the billionth in **10** hrs.

Equation (4) was discovered numerically using integer relation methods over months in our Vancouver lab, **CECM**. It arrived in the coded form:

$$\pi = 4\,{}_2F_1\left(1, \frac{1}{4}; \frac{5}{4}, -\frac{1}{4}\right) + 2\tan^{-1}\left(\frac{1}{2}\right) - \log 5$$

where ${}_2F_1(1, 1/4; 5/4, -1/4) = 0.955933837\ldots$ is a Gauss hypergeometric function.

CARMA

What BBP Is? Reverse Engineered Mathematics

This is based on the following then new formula for π:

$$\pi = \sum_{i=0}^{\infty} \frac{1}{16^i} \left(\frac{4}{8i+1} - \frac{2}{8i+4} - \frac{1}{8i+5} - \frac{1}{8i+6} \right) \qquad (4)$$

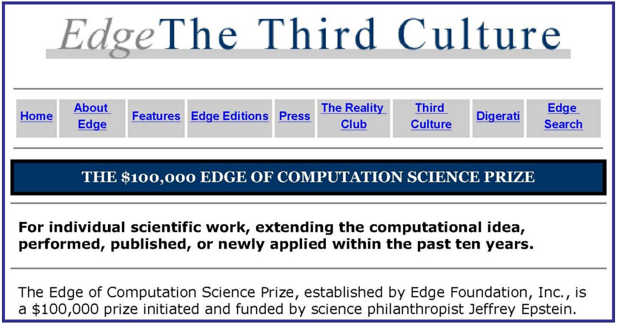
- The millionth hex digit (four millionth binary digit) of π can be found in under **30** secs on a fairly new computer in Maple (not C++) and the billionth in **10** hrs.

Equation (4) was discovered numerically using integer relation methods over months in our Vancouver lab, **CECM**. It arrived in the coded form:

$$\pi = 4\,{}_2F_1\left(1, \frac{1}{4}; \frac{5}{4}, -\frac{1}{4}\right) + 2\tan^{-1}\left(\frac{1}{2}\right) - \log 5$$

where ${}_2F_1(1, 1/4; 5/4, -1/4) = 0.955933837\ldots$ is a Gauss hypergeometric function.

Edge of Computation Prize Finalist


EdgeThe Third Culture

Home | About Edge | Features | Edge Editions | Press | The Reality Club | Third Culture | Digerati | Edge Search

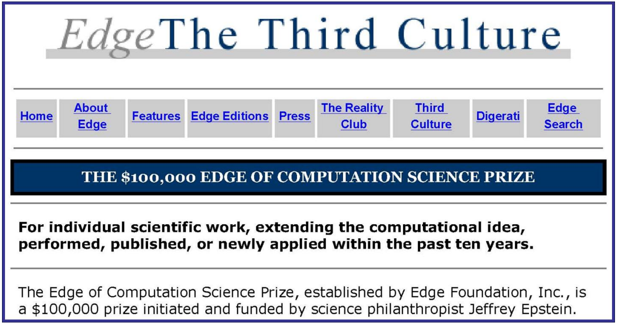
THE $100,000 EDGE OF COMPUTATION SCIENCE PRIZE

For individual scientific work, extending the computational idea, performed, published, or newly applied within the past ten years.

The Edge of Computation Science Prize, established by Edge Foundation, Inc., is a $100,000 prize initiated and funded by science philanthropist Jeffrey Epstein.

- BBP was the only mathematical finalist (of about 40) for the first **Edge of Computation Science Prize**
  - Along with founders of Google, Netscape, Celera and many brilliant thinkers, ...
- Won by David Deutsch — discoverer of Quantum Computing.

Edge of Computation Prize Finalist



- BBP was the only mathematical finalist (of about 40) for the first **Edge of Computation Science Prize**
  - Along with founders of Google, Netscape, Celera and many brilliant thinkers, ...
- Won by David Deutsch — discoverer of Quantum Computing.

Edge of Computation Prize Finalist

Edge The Third Culture

Home | About Edge | Features | Edge Editions | Press | The Reality Club | Third Culture | Digerati | Edge Search

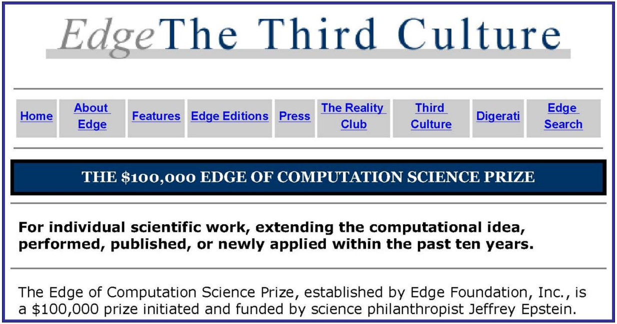
THE $100,000 EDGE OF COMPUTATION SCIENCE PRIZE

For individual scientific work, extending the computational idea, performed, published, or newly applied within the past ten years.

The Edge of Computation Science Prize, established by Edge Foundation, Inc., is a $100,000 prize initiated and funded by science philanthropist Jeffrey Epstein.

- BBP was the only mathematical finalist (of about 40) for the first **Edge of Computation Science Prize**
  - Along with founders of Google, Netscape, Celera and many brilliant thinkers, ...
- Won by David Deutsch — discoverer of Quantum Computing.

Edge of Computation Prize Finalist

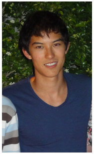


- BBP was the only mathematical finalist (of about 40) for the first **Edge of Computation Science Prize**
  - Along with founders of Google, Netscape, Celera and many brilliant thinkers, ...
- Won by David Deutsch — discoverer of Quantum Computing.

CARMA

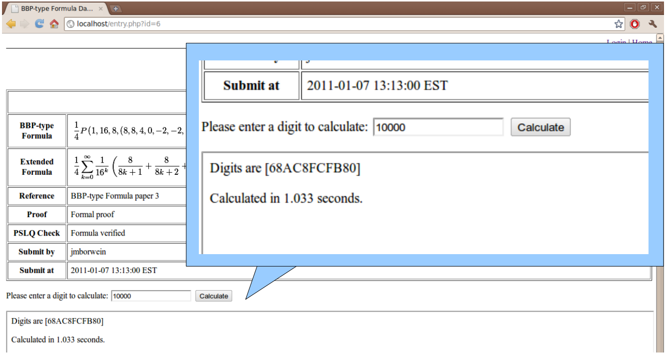
BBP Formula Database http://carma.newcastle.edu.au/bbp

SKIP



Matthew Tam has built an interactive website.

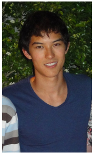
1. It includes most known BBP formulas.
2. It allows digit computation, is searchable, updatable and more.



CARMA

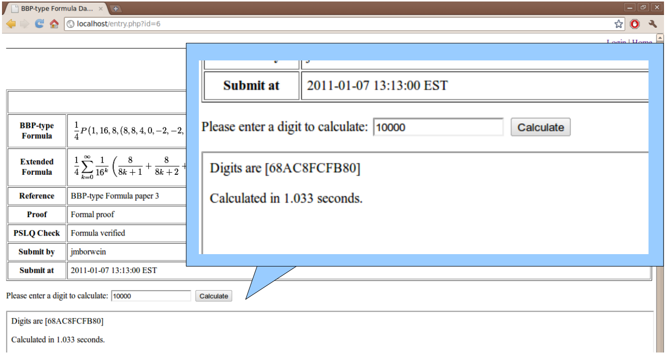
BBP Formula Database http://carma.newcastle.edu.au/bbp

▸ SKIP



Matthew Tam has built an interactive website.

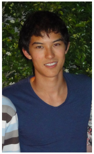
1. It includes most known BBP formulas.
2. It allows digit computation, is searchable, updatable and more.



CARMA

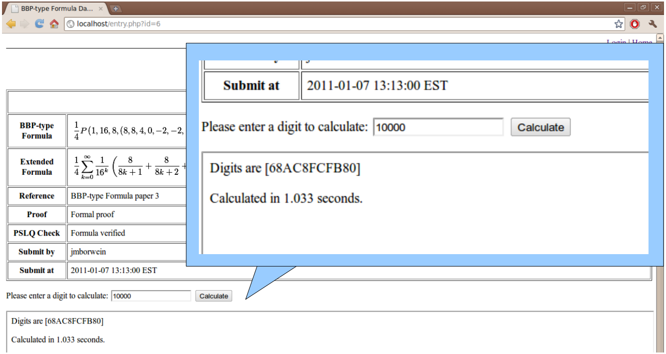
BBP Formula Database http://carma.newcastle.edu.au/bbp

▸ SKIP



Matthew Tam has built an interactive website.

1. It includes most known BBP formulas.
2. It allows digit computation, is searchable, updatable and more.



Mathematical Interlude: (Maple, Mathematica and Human)

Proof of (4). For $0 < k < 8$,

$$\int_0^{1/\sqrt{2}} \frac{x^{k-1}}{1-x^8}\,dx = \int_0^{1/\sqrt{2}} \sum_{i=0}^{\infty} x^{k-1+8i}\,dx = \frac{1}{2^{k/2}} \sum_{i=0}^{\infty} \frac{1}{16^i(8i+k)}.$$

Thus, one can write

$$\sum_{i=0}^{\infty} \frac{1}{16^i}\left(\frac{4}{8i+1} - \frac{2}{8i+4} - \frac{1}{8i+5} - \frac{1}{8i+6}\right)$$

$$= \int_0^{1/\sqrt{2}} \frac{4\sqrt{2} - 8x^3 - 4\sqrt{2}x^4 - 8x^5}{1-x^8}\,dx,$$

which on substituting $y := \sqrt{2}x$ becomes

$$\int_0^1 \frac{16\,y - 16}{y^4 - 2\,y^3 + 4\,y - 4}\,dy = \int_0^1 \frac{4y}{y^2-2}\,dy - \int_0^1 \frac{4y-8}{y^2-2y+2}\,dy = \pi.$$

CARMA

QED

Mathematical Interlude: (Maple, Mathematica and Human)

Proof of (4). For $0 < k < 8$,

$$\int_0^{1/\sqrt{2}} \frac{x^{k-1}}{1-x^8}\,dx = \int_0^{1/\sqrt{2}} \sum_{i=0}^{\infty} x^{k-1+8i}\,dx = \frac{1}{2^{k/2}} \sum_{i=0}^{\infty} \frac{1}{16^i(8i+k)}.$$

Thus, one can write

$$\sum_{i=0}^{\infty} \frac{1}{16^i}\left(\frac{4}{8i+1} - \frac{2}{8i+4} - \frac{1}{8i+5} - \frac{1}{8i+6}\right)$$
$$= \int_0^{1/\sqrt{2}} \frac{4\sqrt{2} - 8x^3 - 4\sqrt{2}x^4 - 8x^5}{1-x^8}\,dx,$$

which on substituting $y := \sqrt{2}x$ becomes

$$\int_0^1 \frac{16\,y - 16}{y^4 - 2\,y^3 + 4\,y - 4}\,dy = \int_0^1 \frac{4y}{y^2-2}\,dy - \int_0^1 \frac{4y-8}{y^2-2y+2}\,dy = \pi.$$

QED

Mathematical Interlude: (Maple, Mathematica and Human)

Proof of (4). For $0 < k < 8$,

$$\int_0^{1/\sqrt{2}} \frac{x^{k-1}}{1-x^8}\,dx = \int_0^{1/\sqrt{2}} \sum_{i=0}^{\infty} x^{k-1+8i}\,dx = \frac{1}{2^{k/2}} \sum_{i=0}^{\infty} \frac{1}{16^i(8i+k)}.$$

Thus, one can write

$$\sum_{i=0}^{\infty} \frac{1}{16^i}\left(\frac{4}{8i+1} - \frac{2}{8i+4} - \frac{1}{8i+5} - \frac{1}{8i+6}\right)$$
$$= \int_0^{1/\sqrt{2}} \frac{4\sqrt{2} - 8x^3 - 4\sqrt{2}x^4 - 8x^5}{1-x^8}\,dx,$$

which on substituting $y := \sqrt{2}x$ becomes

$$\int_0^1 \frac{16\,y - 16}{y^4 - 2\,y^3 + 4\,y - 4}\,dy = \int_0^1 \frac{4y}{y^2-2}\,dy - \int_0^1 \frac{4y-8}{y^2-2y+2}\,dy = \pi.$$

QED

Tuning BBP Computation

- **1997**. Fabrice Bellard of INRIA computed 152 bits of π starting at the trillionth position;
  – in 12 days on 20 workstations working in parallel over the Internet.

Bellard used the following variant of (4):

$$\pi = 4\sum_{k=0}^{\infty}\frac{(-1)^k}{4^k(2k+1)} - \frac{1}{64}\sum_{k=0}^{\infty}\frac{(-1)^k}{1024^k}\left(\frac{32}{4k+1}+\frac{8}{4k+2}+\frac{1}{4k+3}\right) \quad (5)$$

This frequently-used formula is a little faster than (4).

Colin Percival **(L)** and Fabrice Bellard **(R)**

CARMA

Tuning BBP Computation

- **1997**. Fabrice Bellard of INRIA computed 152 bits of π starting at the trillionth position;
  - in 12 days on 20 workstations working in parallel over the Internet.

Bellard used the following variant of (4):

$$\pi = 4\sum_{k=0}^{\infty}\frac{(-1)^k}{4^k(2k+1)} - \frac{1}{64}\sum_{k=0}^{\infty}\frac{(-1)^k}{1024^k}\left(\frac{32}{4k+1}+\frac{8}{4k+2}+\frac{1}{4k+3}\right) \quad (5)$$

This frequently-used formula is a little faster than (4).

Colin Percival **(L)** and Fabrice Bellard **(R)**

Hexadecimal Digits

1998. Colin Percival, a 17-year-old at Simon Fraser, found the five trillionth and ten trillionth hex digits on 25 machines.

2000. He then found the quadrillionth binary digit is **0**.

- He used **250 CPU-years, on 1734 machines in 56 countries.**
- The largest calculation ever done before **Toy Story Two**.

Position	Hex Digits
10^6	26C65E52CB4593
10^7	17AF5863EFED8D
10^8	ECB840E21926EC
10^9	85895585A0428B
10^{10}	921C73C6838FB2
10^{11}	9C381872D27596
1.25×10^{12}	07E45733CC790B
2.5×10^{14}	E6216B069CB6C1

Hexadecimal Digits

1998. Colin Percival, a 17-year-old at Simon Fraser, found the five trillionth and ten trillionth hex digits on 25 machines.

2000. He then found the quadrillionth binary digit is **0**.

- He used **250 CPU-years, on 1734 machines in 56 countries**.
- The largest calculation ever done before **Toy Story Two**.

Position	Hex Digits
10^6	26C65E52CB4593
10^7	17AF5863EFED8D
10^8	ECB840E21926EC
10^9	85895585A0428B
10^{10}	921C73C6838FB2
10^{11}	9C381872D27596
1.25×10^{12}	07E45733CC790B
2.5×10^{14}	E6216B069CB6C1

Hexadecimal Digits

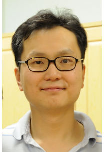
1998. Colin Percival, a 17-year-old at Simon Fraser, found the five trillionth and ten trillionth hex digits on 25 machines.
2000. He then found the quadrillionth binary digit is **0**.

- He used **250 CPU-years, on 1734 machines in 56 countries**.
- The largest calculation ever done before **Toy Story Two**.

Position	Hex Digits
10^6	26C65E52CB4593
10^7	17AF5863EFED8D
10^8	ECB840E21926EC
10^9	85895585A0428B
10^{10}	921C73C6838FB2
10^{11}	9C381872D27596
1.25×10^{12}	07E45733CC790B
2.5×10^{14}	E6216B069CB6C1

Everything Doubles Eventually

SKIP



July 2010. Tsz-Wo Sz of Yahoo!/Cloud computing found the two quadrillionth **bit**. The computation took **23** real days and **503 CPU years**; and involved as many as **4000 machines**.

Abstract

We present a new record on computing specific bits of π, the mathematical constant, and discuss performing such computations on Apache Hadoop clusters. The new record represented in hexadecimal is

```
0 E6C1294A ED40403F 56D2D764 026265BC A98511D0
FCFFAA10 F4D28B1B B5392B8
```

which has **256 bits** ending at the $2,000,000,000,000,000,252^{th}$ bit position. The position of the first bit is $1,999,999,999,999,997$ and the value of the two quadrillionth bit is 0.

Everything Doubles Eventually

▸ SKIP

July 2010. Tsz-Wo Sz of Yahoo!/Cloud computing found the two quadrillionth **bit**. The computation took **23** real days and **503 CPU years**; and involved as many as **4000 machines**.

Abstract

We present a new record on computing specific bits of π, the mathematical constant, and discuss performing such computations on Apache Hadoop clusters. The new record represented in hexadecimal is

```
0 E6C1294A ED40403F 56D2D764 026265BC A98511D0
FCFFAA10 F4D28B1B B5392B8
```

which has **256 bits** ending at the $2,000,000,000,000,000,252^{th}$ bit position. The position of the first bit is $1,999,999,999,999,997$ and the value of the two quadrillionth bit is 0.

BBP Formulas Explained

Base-b BBP numbers are constants of the form

$$\alpha = \sum_{k=0}^{\infty} \frac{p(k)}{q(k)b^k}, \tag{6}$$

where $p(k)$ and $q(k)$ are integer polynomials and $b = 2, 3, \ldots$.

- I illustrate why this works in binary for $\log 2$. We start with:

$$\log 2 = \sum_{k=0}^{\infty} \frac{1}{k2^k} \tag{7}$$

as discovered by Euler.

- We wish to compute digits *beginning* at position $d+1$.
- Equivalently, we need $\{2^d \log 2\}$ ($\{\cdot\}$ is the fractional part).

CARMA

BBP Formulas Explained

Base-b BBP numbers are constants of the form

$$\alpha = \sum_{k=0}^{\infty} \frac{p(k)}{q(k)b^k}, \tag{6}$$

where $p(k)$ and $q(k)$ are integer polynomials and $b = 2, 3, \ldots$.

- I illustrate why this works in binary for $\log 2$. We start with:

$$\log 2 = \sum_{k=0}^{\infty} \frac{1}{k2^k} \tag{7}$$

as discovered by Euler.

- We wish to compute digits *beginning* at position $d+1$.
- Equivalently, we need $\{2^d \log 2\}$ ($\{\cdot\}$ is the fractional part).

BBP Formulas Explained

Base-b BBP numbers are constants of the form

$$\alpha = \sum_{k=0}^{\infty} \frac{p(k)}{q(k)b^k}, \quad (6)$$

where $p(k)$ and $q(k)$ are integer polynomials and $b = 2, 3, \ldots$.

- I illustrate why this works in binary for $\log 2$. We start with:

$$\log 2 = \sum_{k=0}^{\infty} \frac{1}{k2^k} \quad (7)$$

as discovered by Euler.

- We wish to compute digits *beginning* at position $d+1$.
- Equivalently, we need $\{2^d \log 2\}$ ($\{\cdot\}$ is the fractional part).

BBP Formula for $\log 2$

We can write

$$\begin{aligned}\{2^d \log 2\} &= \left\{\left\{\sum_{k=0}^{d} \frac{2^{d-k}}{k}\right\} + \left\{\sum_{k=d+1}^{\infty} \frac{2^{d-k}}{k}\right\}\right\}\\ &= \left\{\left\{\sum_{k=0}^{d} \frac{\mathbf{2^{d-k}} \bmod \mathbf{k}}{k}\right\} + \left\{\sum_{k=d+1}^{\infty} \frac{\mathbf{2^{d-k}}}{\mathbf{k}}\right\}\right\}. \qquad (8)\end{aligned}$$

- The key: the numerator in (8), $2^{d-k} \bmod k$, can be found rapidly by binary exponentiation, performed modulo k. So,

$$3^{17} = ((((3^2)^2)^2)^2) \cdot 3$$

uses only **5** multiplications, not the usual **16**. Moreover, $3^{17} \bmod 10$ is done as $3^2 = 9; 9^2 = 1; 1^2 = 1; 1^2 = 1; 1 \times 3 = 3$.

CARMA

BBP Formula for $\log 2$

We can write

$$\begin{aligned} \{2^d \log 2\} &= \left\{\left\{\sum_{k=0}^{d} \frac{2^{d-k}}{k}\right\} + \left\{\sum_{k=d+1}^{\infty} \frac{2^{d-k}}{k}\right\}\right\} \\ &= \left\{\left\{\sum_{k=0}^{d} \frac{\mathbf{2^{d-k}} \bmod \mathbf{k}}{k}\right\} + \left\{\sum_{k=d+1}^{\infty} \frac{\mathbf{2^{d-k}}}{\mathbf{k}}\right\}\right\}. \qquad (8) \end{aligned}$$

- The key: the numerator in (8), $2^{d-k} \bmod k$, can be found rapidly by binary exponentiation, performed modulo k. So,

$$3^{17} = ((((3^2)^2)^2)^2) \cdot 3$$

uses only **5** multiplications, not the usual **16**. Moreover, $3^{17} \bmod 10$ is done as $3^2 = 9; 9^2 = 1; 1^2 = 1; 1^2 = 1; 1 \times 3 = 3$.

CARMA

BBP Formula for $\log 2$

We can write

$$\begin{aligned}\{2^d \log 2\} &= \left\{\left\{\sum_{k=0}^{d} \frac{2^{d-k}}{k}\right\} + \left\{\sum_{k=d+1}^{\infty} \frac{2^{d-k}}{k}\right\}\right\} \\ &= \left\{\left\{\sum_{k=0}^{d} \frac{\mathbf{2^{d-k}} \bmod \mathbf{k}}{k}\right\} + \left\{\sum_{k=d+1}^{\infty} \frac{\mathbf{2^{d-k}}}{\mathbf{k}}\right\}\right\}. \qquad (8)\end{aligned}$$

- The key: the numerator in (8), $2^{d-k} \bmod k$, can be found rapidly by binary exponentiation, performed modulo k. So,

$$3^{17} = ((((3^2)^2)^2)^2) \cdot 3$$

uses only **5** multiplications, not the usual **16**. Moreover, $3^{17} \bmod 10$ is done as $3^2 = 9; 9^2 = 1; 1^2 = 1; 1^2 = 1; 1 \times 3 = 3$

Catalan's Constant G: and BBP for G in Binary

The simplest number not proven irrational is

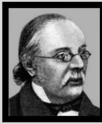
$$G := 1 - \frac{1}{3^2} + \frac{1}{5^2} - \frac{1}{7^2} + \cdots, \qquad \frac{\pi^2}{12} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots$$

2009. G is calculated to **31.026** billion digits. Records often use:

$$G = \frac{3}{8}\sum_{n=0}^{\infty} \frac{1}{\binom{2n}{n}(2n+1)^2} + \frac{\pi}{8}\log(2+\sqrt{3}) \text{ (Ramanujan)} \tag{9}$$

– holds since $G = -T(\frac{\pi}{4}) = -\frac{3}{2}T(\frac{\pi}{12})$ where $T(\theta) := \int_0^\theta \log\tan\sigma\, d\sigma$.

– An **18** term binary BBP formula for $G = 0.9159655941772190\ldots$ is:



$$G = \cfrac{1}{1 + \cfrac{1}{10 + \cfrac{1}{1 + \cfrac{1}{8 + \cfrac{1}{1 + \cfrac{1}{88 + \ldots}}}}}}$$

$$G = \sum_{k=0}^{\infty} \frac{1}{4^{6k+5}} \left(\frac{3072}{(24k+1)^2} - \frac{3072}{(24k+2)^2} - \frac{23040}{(24k+3)^2} + \frac{12288}{(24k+4)^2} - \frac{768}{(24k+5)^2} + \frac{9216}{(24k+6)^2} + \frac{10368}{(24k+8)^2} + \frac{2496}{(24k+9)^2} - \frac{192}{(24k+10)^2} + \frac{768}{(24k+12)^2} - \frac{48}{(24k+13)^2} + \frac{360}{(24k+15)^2} + \frac{648}{(24k+16)^2} + \frac{12}{(24k+17)^2} + \frac{168}{(24k+18)^2} + \frac{48}{(24k+20)^2} - \frac{39}{(24k+21)^2} \right)$$

Eugene Catalan (**1818-94**)– a revolutionary

Catalan's Constant G: and BBP for G in Binary

The simplest number not proven irrational is

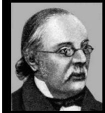
$$G := 1 - \frac{1}{3^2} + \frac{1}{5^2} - \frac{1}{7^2} + \cdots, \qquad \frac{\pi^2}{12} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots$$

2009. G is calculated to **31.026** billion digits. Records often use:

$$G = \frac{3}{8}\sum_{n=0}^{\infty} \frac{1}{\binom{2n}{n}(2n+1)^2} + \frac{\pi}{8}\log(2+\sqrt{3}) \text{ (Ramanujan)} \qquad (9)$$

– holds since $G = -T(\frac{\pi}{4}) = -\frac{3}{2}T(\frac{\pi}{12})$ where $T(\theta) := \int_0^\theta \log\tan\sigma\, d\sigma$.

– An **18** term binary BBP formula for $G = 0.9159655941772190\ldots$ is:



$$G = \cfrac{1}{1 + \cfrac{1}{10 + \cfrac{1}{1 + \cfrac{1}{8 + \cfrac{1}{1 + \cfrac{1}{88 + \ldots}}}}}}$$

$$G = \sum_{k=0}^{\infty} \frac{1}{4^{6k+5}} \left(\frac{3072}{(24k+1)^2} - \frac{3072}{(24k+2)^2} - \frac{23040}{(24k+3)^2} + \frac{12288}{(24k+4)^2} - \frac{768}{(24k+5)^2} + \frac{9216}{(24k+6)^2} + \frac{10368}{(24k+8)^2} + \frac{2496}{(24k+9)^2} - \frac{192}{(24k+10)^2} + \frac{768}{(24k+12)^2} - \frac{48}{(24k+13)^2} + \frac{360}{(24k+15)^2} + \frac{648}{(24k+16)^2} + \frac{12}{(24k+17)^2} + \frac{168}{(24k+18)^2} + \frac{48}{(24k+20)^2} - \frac{39}{(24k+21)^2} \right)$$

Eugene Catalan (**1818-94**)– a revolutionary

Catalan's Constant G: and BBP for G in Binary

The simplest number not proven irrational is

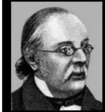
$$G := 1 - \frac{1}{3^2} + \frac{1}{5^2} - \frac{1}{7^2} + \cdots, \qquad \frac{\pi^2}{12} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots$$

2009. G is calculated to **31.026** billion digits. Records often use:

$$G = \frac{3}{8} \sum_{n=0}^{\infty} \frac{1}{\binom{2n}{n}(2n+1)^2} + \frac{\pi}{8} \log(2+\sqrt{3}) \quad \text{(Ramanujan)} \qquad (9)$$

– holds since $G = -T(\frac{\pi}{4}) = -\frac{3}{2} T(\frac{\pi}{12})$ where $T(\theta) := \int_0^\theta \log \tan \sigma \, d\sigma$.

– An **18** term binary BBP formula for $G = 0.9159655941772190\ldots$ is:



$$G = \cfrac{1}{1 + \cfrac{1}{10 + \cfrac{1}{1 + \cfrac{1}{8 + \cfrac{1}{1 + \cfrac{1}{88 + \ldots}}}}}}$$

$$G = \sum_{k=0}^{\infty} \frac{1}{4^{6k+5}} \left(\frac{3072}{(24k+1)^2} - \frac{3072}{(24k+2)^2} - \frac{23040}{(24k+3)^2} + \frac{12288}{(24k+4)^2} - \frac{768}{(24k+5)^2} + \frac{9216}{(24k+6)^2} + \frac{10368}{(24k+8)^2} + \frac{2496}{(24k+9)^2} - \frac{192}{(24k+10)^2} + \frac{768}{(24k+12)^2} - \frac{48}{(24k+13)^2} + \frac{360}{(24k+15)^2} + \frac{648}{(24k+16)^2} + \frac{12}{(24k+17)^2} + \frac{168}{(24k+18)^2} + \frac{48}{(24k+20)^2} - \frac{39}{(24k+21)^2} \right)$$

CARMA

Eugene Catalan (**1818-94**)– a revolutionary

Catalan's Constant G: and BBP for G in Binary

The simplest number not proven irrational is

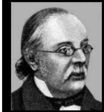
$$G := 1 - \frac{1}{3^2} + \frac{1}{5^2} - \frac{1}{7^2} + \cdots, \qquad \frac{\pi^2}{12} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots$$

2009. G is calculated to **31.026** billion digits. Records often use:

$$G = \frac{3}{8} \sum_{n=0}^{\infty} \frac{1}{\binom{2n}{n}(2n+1)^2} + \frac{\pi}{8} \log(2+\sqrt{3}) \text{ (Ramanujan)} \tag{9}$$

– holds since $G = -T(\frac{\pi}{4}) = -\frac{3}{2} T(\frac{\pi}{12})$ where $T(\theta) := \int_0^\theta \log \tan \sigma d\sigma$.

– An **18** term binary BBP formula for $G = 0.9159655941772190\ldots$ is:



$$G = \cfrac{1}{1 + \cfrac{1}{10 + \cfrac{1}{1 + \cfrac{1}{8 + \cfrac{1}{1 + \cfrac{1}{88 + \ldots}}}}}}$$

$$G = \sum_{k=0}^{\infty} \frac{1}{4^{6k+5}} \left(\frac{3072}{(24k+1)^2} - \frac{3072}{(24k+2)^2} - \frac{23040}{(24k+3)^2} + \frac{12288}{(24k+4)^2} - \frac{768}{(24k+5)^2} + \frac{9216}{(24k+6)^2} + \frac{10368}{(24k+8)^2} + \frac{2496}{(24k+9)^2} - \frac{192}{(24k+10)^2} + \frac{768}{(24k+12)^2} - \frac{48}{(24k+13)^2} + \frac{360}{(24k+15)^2} + \frac{648}{(24k+16)^2} + \frac{12}{(24k+17)^2} + \frac{168}{(24k+18)^2} + \frac{48}{(24k+20)^2} - \frac{39}{(24k+21)^2} \right)$$

CARMA

Eugene Catalan (**1818-94**)– a revolutionary

Catalan's Constant G: and BBP for G in Binary

The simplest number not proven irrational is

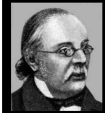
$$G := 1 - \frac{1}{3^2} + \frac{1}{5^2} - \frac{1}{7^2} + \cdots, \qquad \frac{\pi^2}{12} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots$$

2009. G is calculated to **31.026** billion digits. Records often use:

$$G = \frac{3}{8} \sum_{n=0}^{\infty} \frac{1}{\binom{2n}{n}(2n+1)^2} + \frac{\pi}{8} \log(2+\sqrt{3}) \text{ (Ramanujan)} \tag{9}$$

- holds since $G = -T(\frac{\pi}{4}) = -\frac{3}{2}\,T(\frac{\pi}{12})$ where $T(\theta) := \int_0^\theta \log\tan\sigma\, d\sigma$.

- An **18** term binary BBP formula for $G = 0.9159655941772190\ldots$ is:



$$G = \cfrac{1}{1 + \cfrac{1}{10 + \cfrac{1}{1 + \cfrac{1}{8 + \cfrac{1}{1 + \cfrac{1}{88 + \ldots}}}}}}$$

$$G = \sum_{k=0}^{\infty} \frac{1}{4^{6k+5}} \left(\frac{3072}{(24k+1)^2} - \frac{3072}{(24k+2)^2} - \frac{23040}{(24k+3)^2} + \frac{12288}{(24k+4)^2} - \frac{768}{(24k+5)^2} + \frac{9216}{(24k+6)^2} + \frac{10368}{(24k+8)^2} + \frac{2496}{(24k+9)^2} - \frac{192}{(24k+10)^2} + \frac{768}{(24k+12)^2} - \frac{48}{(24k+13)^2} + \frac{360}{(24k+15)^2} + \frac{648}{(24k+16)^2} + \frac{12}{(24k+17)^2} + \frac{168}{(24k+18)^2} + \frac{48}{(24k+20)^2} - \frac{39}{(24k+21)^2} \right)$$

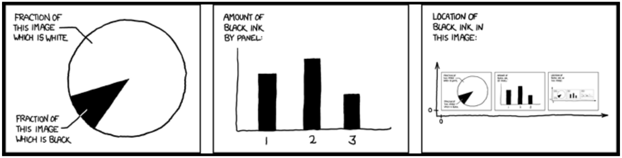
Eugene Catalan (**1818-94**)– a revolutionary

A Better Formula for G

A new **16** term binary formula in concise BBP notation is:

$$G = P(2, \mathbf{4096}, 24, \vec{v}) \quad \text{where}$$
$$\vec{v} := (6144, -6144, -6144, 0, -1536, -3072, -768, 0, -768, -384, 192, 0, -96, 96, 96, 0, 24, 48, 12, 0, 12, 6, -3, 0)$$

It takes almost exactly **8/9**th the time of **18** term formula for G.



- This makes for a very cool calculation
- Since we can not prove G is irrational, *Who can say what might turn up*?

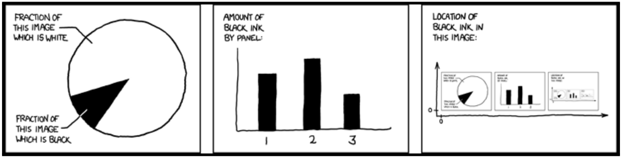
A Better Formula for G

A new **16** term binary formula in concise BBP notation is:

$$G = P\left(2, \mathbf{4096}, 24, \vec{v}\right) \qquad \text{where}$$

$$\vec{v} := (6144, -6144, -6144, 0, -1536, -3072, -768, 0, -768, -384, 192, 0, -96, 96, 96, 0, 24, 48, 12, 0, 12, 6, -3, 0)$$

It takes almost exactly **8/9**th the time of **18** term formula for G.



- This makes for a very cool calculation
- Since we can not prove G is irrational, *Who can say what might turn up*?

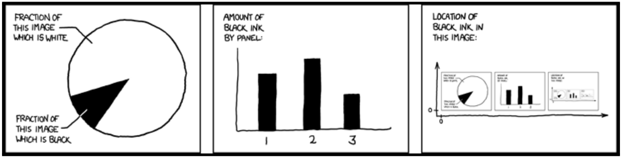
CARMA

A Better Formula for G

A new **16** term binary formula in concise BBP notation is:

$$G = P(2, \mathbf{4096}, 24, \vec{v}) \quad \text{where}$$
$$\vec{v} := (6144, -6144, -6144, 0, -1536, -3072, -768, 0, -768, -384, 192, 0, -96, 96, 96, 0, 24, 48, 12, 0, 12, 6, -3, 0)$$

It takes almost exactly **8/9**th the time of **18** term formula for G.



- This makes for a very cool calculation
- Since we can not prove G is irrational, *Who can say what might turn up*?

CARMA

What About Base Ten?

- The first integer logarithm with no known binary BBP formula is $\log 23$ (since $23 \times 89 = 2^{10} - 1$).

Searches conducted by numerous researchers for base-ten formulas have been unfruitful. Indeed:

2004. D. Borwein (my father), W. Gallway and I showed there are no BBP formulas of the *Machin-type* of (4) for π if base is not a power of **two**.

- Bailey and Crandall have shown connections between the existence of a b-ary BBP formula for α and its base b *normality* (via a dynamical system conjecture).

CARMA

What About Base Ten?

- The first integer logarithm with no known binary BBP formula is $\log 23$ (since $23 \times 89 = 2^{10} - 1$).

Searches conducted by numerous researchers for base-ten formulas have been unfruitful. Indeed:

2004. D. Borwein (my father), W. Gallway and I showed there are no BBP formulas of the *Machin-type* of (4) for π if base is not a power of **two**.

- Bailey and Crandall have shown connections between the existence of a b-ary BBP formula for α and its base b *normality* (via a dynamical system conjecture).

CARMA

What About Base Ten?

- The first integer logarithm with no known binary BBP formula is $\log 23$ (since $23 \times 89 = 2^{10} - 1$).

Searches conducted by numerous researchers for base-ten formulas have been unfruitful. Indeed:

2004. D. Borwein (my father), W. Gallway and I showed there are no BBP formulas of the *Machin-type* of (4) for π if base is not a power of **two**.

- Bailey and Crandall have shown connections between the existence of a b-ary BBP formula for α and its base b *normality* (via a dynamical system conjecture).

What About Base Ten?

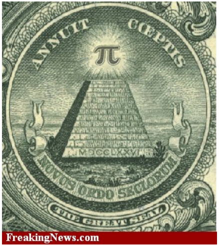
- The first integer logarithm with no known binary BBP formula is $\log 23$ (since $23 \times 89 = 2^{10} - 1$).

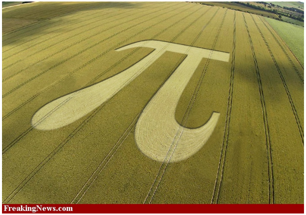
Searches conducted by numerous researchers for base-ten formulas have been unfruitful. Indeed:

2004. D. Borwein (my father), W. Gallway and I showed there are no BBP formulas of the *Machin-type* of (4) for π if base is not a power of **two**.

- Bailey and Crandall have shown connections between the existence of a b-ary BBP formula for α and its base b *normality* (via a dynamical system conjecture).

Pi Photo-shopped: a 2010 PiDay Contest



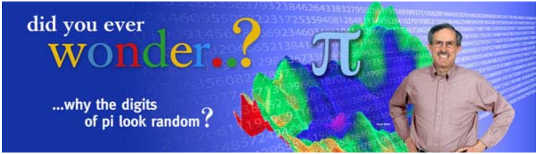


"Noli Credere Pictis"

CARMA

π^2 in Binary and Ternary (unlike π)

Bailey and Pi on a Bus. Only in Berkeley?

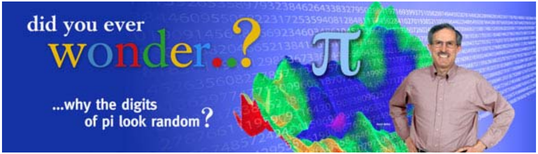


Thanks to Dave Broadhurst, a ternary BBP formula exists for π^2:

$$\pi^2 = \frac{2}{27}\sum_{k=0}^{\infty}\frac{1}{3^{6k}} \times \left\{\frac{243}{(12k+1)^2} - \frac{405}{(12k+2)^2} - \frac{81}{(12k+4)^2} - \frac{27}{(12k+5)^2} - \frac{72}{(12k+6)^2} - \frac{9}{(12k+7)^2} - \frac{9}{(12k+8)^2} - \frac{5}{(12k+10)^2} + \frac{1}{(12k+11)^2}\right\}$$

π^2 in Binary and Ternary (unlike π)

Bailey and Pi on a Bus. Only in Berkeley?



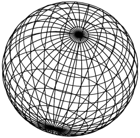
Thanks to Dave Broadhurst, a ternary BBP formula exists for π^2:

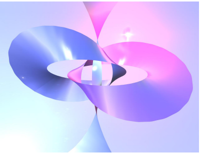
$$\pi^2 = \frac{2}{27}\sum_{k=0}^{\infty}\frac{1}{3^{6k}} \times \left\{\frac{243}{(12k+1)^2} - \frac{405}{(12k+2)^2} - \frac{81}{(12k+4)^2} - \frac{27}{(12k+5)^2} - \frac{72}{(12k+6)^2} - \frac{9}{(12k+7)^2} - \frac{9}{(12k+8)^2} - \frac{5}{(12k+10)^2} + \frac{1}{(12k+11)^2}\right\}$$

A Partner Binary BBP Formula for π^2

$$\pi^2 = \frac{9}{8}\sum_{k=0}^{\infty}\frac{1}{2^{6k}}\left\{\frac{16}{(6k+1)^2}-\frac{24}{(6k+2)^2}-\frac{8}{(6k+3)^2}-\frac{6}{(6k+4)^2}+\frac{1}{(6k+5)^2}\right\}$$

- We do not fully understand why π^2 allows BBP formulas in two distinct bases.





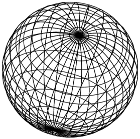
- $2\pi^2$ is the area of a sphere in four-space.
- $\frac{1}{2}\pi^2$ is the volume inside a sphere in four-space (R).
  - So in binary we are computing these fundamental physical constants.

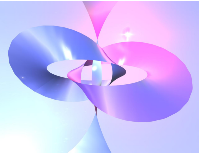
CARMA

A Partner Binary BBP Formula for π^2

$$\pi^2 = \frac{9}{8}\sum_{k=0}^{\infty}\frac{1}{2^{6k}}\left\{\frac{16}{(6k+1)^2}-\frac{24}{(6k+2)^2}-\frac{8}{(6k+3)^2}-\frac{6}{(6k+4)^2}+\frac{1}{(6k+5)^2}\right\}$$

- We do not fully understand why π^2 allows BBP formulas in two distinct bases.



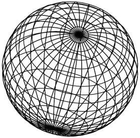


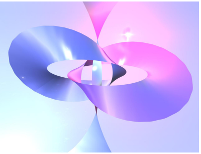
- $2\pi^2$ is the area of a sphere in four-space.
- $\frac{1}{2}\pi^2$ is the volume inside a sphere in four-space (R).
  - So in binary we are computing these fundamental physical constants.

A Partner Binary BBP Formula for π^2

$$\pi^2 = \frac{9}{8}\sum_{k=0}^{\infty}\frac{1}{2^{6k}}\left\{\frac{16}{(6k+1)^2}-\frac{24}{(6k+2)^2}-\frac{8}{(6k+3)^2}-\frac{6}{(6k+4)^2}+\frac{1}{(6k+5)^2}\right\}$$

- We do not fully understand why π^2 allows BBP formulas in two distinct bases.





- $2\pi^2$ is the area of a sphere in four-space.
- $\frac{1}{2}\pi^2$ is the volume inside a sphere in four-space (R).
  - So in binary we are computing these fundamental physical constants.

IBM's New Record Results

Algorithm (What We Did)

Dave Bailey, Andrew Mattingly (L) and Glenn Wightwick (R) of IBM Australia, and I, have obtained and confirmed:

1. **106** digits of π^2 base **2** at the **ten trillion**th place base **64**
2. **94** digits of π^2 base **3** at the **ten trillion**th place base **729**
3. **141** digits of G base **2** at the **ten trillion**th place base **4096**

on a 4-rack BlueGene/P system at IBM's Benchmarking Centre in Rochester, Minn, USA.

How The Australian Reported This

Supercomputer cracks 'impossible' calculation
Jennifer Foreshew
From: The Australian
April 19, 2011 12:00AM

Calculation easy as pi

HUMAN ingenuity and awesome computing power have combined to deliver an algorithm that can identify potential weaknesses in computer system hardware and software.

The BlueGene/P supercomputer system, used for IBM's benchmarking tests and quality control, was used by experts to conquer a calculation thought to be unachievable.

"**It was believed to be impossible until not very long ago that we would ever know the billionth decimal digit of pi**," said Newcastle University laureate professor Jon Borwein.

Professor Borwein, a world-famous mathematical expert, said the computer time spent on the work was equivalent to the time that went into creating a computer-generated movie such as Toy Story 3. "My estimate is that it may be by a factor of three the largest single computation done for any mathematical object ever," he said.

The work would have taken about 1500 years on a single CPU, but it took just a few months of super-computing time. The project was done in conjunction with the Lawrence Berkeley National Laboratory and IBM Australia.

"**What this is driving is a new attack on various classical questions about how random or how complex various bits of math are, and how best to program these things on really large environments with tens or hundreds of thousands of processors**," said Professor Borwein, who is also an expert on pi, the ratio of the circumference of a circle to its diameter, especially its computation.

"If we could prove pi squared was random in some sense then we could use it instead of all the expensive quantum random number generators or pseudo-random number generators that make all of our banking codes safe," he said.

Professor Borwein believes the calculation means more realistic samples could be made.

"We may be able to put some of these algorithms together, mixing this idea of algorithmic randomness with this fairly new area called quantum randomness, using natural processes to build random things," he said.

Professor Borwein hopes a prototype planned for later this year may lead to further advances in the field.

The 3 Records Use Over 1380 CPU Years (135 rack days)

An enormous amount of delicate computation: **1380 years** is a long time. Suppose a spanking new IBM single-core PC went back **1379 years**.

- It would find itself in **632 CE**.
- The year that Mohammed died, and the Caliphate was established. If it then calculated π nonstop:
  - Through the Crusades, black plague, Moguls, Renaissance, discovery of America, Gutenberg, Reformation, invention of steam, Napoleon, electricity, WW2, the transistor, fiber optics,...
- With no breaks or break-downs:
- It would be done next year.

CARMA

The 3 Records Use Over 1380 CPU Years (135 rack days)

An enormous amount of delicate computation: **1380 years** is a long time. Suppose a spanking new IBM single-core PC went back **1379 years**.

- It would find itself in **632 CE**.
- The year that Mohammed died, and the Caliphate was established. If it then calculated π nonstop:
  - Through the Crusades, black plague, Moguls, Renaissance, discovery of America, Gutenberg, Reformation, invention of steam, Napoleon, electricity, WW2, the transistor, fiber optics,...
- With no breaks or break-downs:
- It would be done next year.

The 3 Records Use Over 1380 CPU Years (135 rack days)

An enormous amount of delicate computation: **1380 years** is a long time. Suppose a spanking new IBM single-core PC went back **1379 years**.

- It would find itself in **632 CE**.
- The year that Mohammed died, and the Caliphate was established. If it then calculated π nonstop:
    - Through the Crusades, black plague, Moguls, Renaissance, discovery of America, Gutenberg, Reformation, invention of steam, Napoleon, electricity, WW2, the transistor, fiber optics,...
- With no breaks or break-downs:
- It would be done next year.

CARMA

The 3 Records Use Over 1380 CPU Years (135 rack days)

An enormous amount of delicate computation: **1380 years** is a long time. Suppose a spanking new IBM single-core PC went back **1379 years**.

- It would find itself in **632 CE**.
- The year that Mohammed died, and the Caliphate was established. If it then calculated π nonstop:
  - Through the Crusades, black plague, Moguls, Renaissance, discovery of America, Gutenberg, Reformation, invention of steam, Napoleon, electricity, WW2, the transistor, fiber optics,...
- With no breaks or break-downs:
- It would be done next year.

CARMA

The 3 Records Use Over 1380 CPU Years (135 rack days)

An enormous amount of delicate computation: **1380 years** is a long time. Suppose a spanking new IBM single-core PC went back **1379 years**.

- It would find itself in **632 CE**.
- The year that Mohammed died, and the Caliphate was established. If it then calculated π nonstop:
  - Through the Crusades, black plague, Moguls, Renaissance, discovery of America, Gutenberg, Reformation, invention of steam, Napoleon, electricity, WW2, the transistor, fiber optics,...
- With no breaks or break-downs:
- It would be done next year.

CARMA

The 3 Records Use Over 1380 CPU Years (135 rack days)

An enormous amount of delicate computation: **1380 years** is a long time. Suppose a spanking new IBM single-core PC went back **1379 years**.

- It would find itself in **632 CE**.
- The year that Mohammed died, and the Caliphate was established. If it then calculated π nonstop:
  - Through the Crusades, black plague, Moguls, Renaissance, discovery of America, Gutenberg, Reformation, invention of steam, Napoleon, electricity, WW2, the transistor, fiber optics,...

- With no breaks or break-downs:
- It would be done next year.

CARMA

The 3 Records Use Over 1380 CPU Years (135 rack days)

An enormous amount of delicate computation: **1380 years** is a long time. Suppose a spanking new IBM single-core PC went back **1379 years**.

- It would find itself in **632 CE**.
- The year that Mohammed died, and the Caliphate was established. If it then calculated π nonstop:
  - Through the Crusades, black plague, Moguls, Renaissance, discovery of America, Gutenberg, Reformation, invention of steam, Napoleon, electricity, WW2, the transistor, fiber optics,...

- With no breaks or break-downs:
- It would be done next year.

CARMA

The 3 Records Use Over 1380 CPU Years (135 rack days)

An enormous amount of delicate computation: **1380 years** is a long time. Suppose a spanking new IBM single-core PC went back **1379 years**.

- It would find itself in **632 CE**.
- The year that Mohammed died, and the Caliphate was established. If it then calculated π nonstop:
  - Through the Crusades, black plague, Moguls, Renaissance, discovery of America, Gutenberg, Reformation, invention of steam, Napoleon, electricity, WW2, the transistor, fiber optics,...
- With no breaks or break-downs:
- It would be done next year.

CARMA

IBM's New Results: π^2 base 2

Algorithm (10 trillionth digits of π^2 in base 64 — in **230** years)

1. The calculation took, on average, **253529** seconds per thread. It was broken into 7 "partitions" of **2048** threads each. For a total of $7 \cdot 2048 \cdot 253529 = 3.6 \cdot 10^9$ CPU seconds.

2. On a single Blue Gene/P CPU it *would* take **115 years**!

   Each rack of BG/P contains 4096 threads (or cores). Thus, we used $\frac{7 \cdot 2048 \cdot 253529}{4096 \cdot 60 \cdot 60 \cdot 24} =$ **10.3** "rack days".

3. The verification run took the same time (within a few minutes): **106 base 2 digits** are in agreement.

```
base-8 digits = 75|601145053032364757245000057432627545303630524163506 34|573227604
                   601145053032364757245000057432627545303630524163506 34|22021056612
```

CARMA

IBM's New Results: π^2 base **3**

Algorithm (10 trillionth digits of π^2 in base 729 — in **414** years)

1. The calculation took, on average, **795773** seconds per thread.
   It was broken into 4 "partitions" of **2048** threads each.
   For a total of $4 \cdot 2048 \cdot 795773 = 6.5 \cdot 10^9$ CPU seconds.

2. On a single Blue Gene/P CPU it *would* take **207 years**!

   Each rack of BG/P contains 4096 threads (or cores).
   Thus, we used $\frac{4 \cdot 2048 \cdot 795773}{4096 \cdot 60 \cdot 60 \cdot 24} =$ **18.4** "rack days".

3. The verification run took the same time (within a few minutes): **94 base 3 digits** are in agreement.

```
base-9 digits = 001|12264485064548583177111135210162856048323453468|10565567|635862
                   12264485064548583177111135210162856048323453468|04744867|134524345
```

CARMA

IBM's New Results: G base 2

Algorithm (10 trillionth digits of G in base **4096** — in **735** years)

1. The calculation took, on average, **707857** seconds per thread. It was broken into 8 "partitions" of **2048** threads each. For a total of $8 \cdot 2048 \cdot 707857 = 1.2 \cdot 10^{10}$ CPU seconds.
2. On a single Blue Gene/P CPU it *would* take **368 years**!

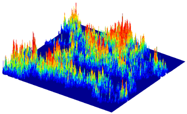
   Each rack of BG/P contains 4096 threads (or cores). Thus, we used $\frac{8 \cdot 2048 \cdot 707857}{4096 \cdot 60 \cdot 60 \cdot 24}$ = **32.8** "rack days".
3. The verification run took the same time (within a few minutes): **141 base 2 digits** were in agreement.

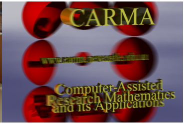
```
base-8 digits = 76|3470505377477705112261337162012525732721732452|6000177545727
                   3470505377477705112261337162012525732721732452|5703510516602536
```

CARMA

4. Animation, Simulation and Stereo . . .

The latest developments in computer and video technology have provided a multiplicity of computational and symbolic tools that have rejuvenated mathematics and mathematics education. Two important examples of this revitalization are experimental mathematics and visual theorems — ICMI Study 19



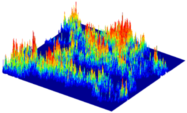


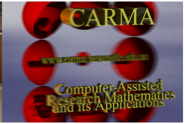
Cinderella, 3.14 min of Pi, Catalan's constant and **Passive Three D**

4. Animation, Simulation and Stereo . . .

> *The latest developments in computer and video technology have provided a multiplicity of computational and symbolic tools that have rejuvenated mathematics and mathematics education. Two important examples of this revitalization are experimental mathematics and visual theorems* — ICMI Study 19





Cinderella, 3.14 min of Pi, Catalan's constant and **Passive Three D**

Thank You to All: Family, Mentors, Colleagues, Students

Related Material (in press):

1. Divide and Concur: www.carma.newcastle.edu.au/jon/dr-jmm11.pptx
2. Walks and Measures: www.carma.newcastle.edu.au/jon/nist-handbook.pdf
3. Pi Day 2011: carma.newcastle.edu.au/jon/piday.pdf
4. BBP and Blue Gene: www.carma.newcastle.edu.au/jon/bbp-bluegene.pdf

2010: Communication is not yet always perfect

Thank You to All: Family, Mentors, Colleagues, Students

Related Material (in press):

1. Divide and Concur:
   www.carma.newcastle.edu.au/jon/dr-jmm11.pptx
2. Walks and Measures:
   www.carma.newcastle.edu.au/jon/nist-handbook.pdf
3. Pi Day 2011:
   carma.newcastle.edu.au/jon/piday.pdf
4. BBP and Blue Gene:
   www.carma.newcastle.edu.au/jon/bbp-bluegene.pdf

2010: Communication is not yet always perfect

Thank You to All: Family, Mentors, Colleagues, Students

Related Material (in press):

1. Divide and Concur:
   www.carma.newcastle.edu.au/jon/dr-jmm11.pptx
2. Walks and Measures:
   www.carma.newcastle.edu.au/jon/nist-handbook.pdf
3. Pi Day 2011:
   carma.newcastle.edu.au/jon/piday.pdf
4. BBP and Blue Gene:
   www.carma.newcastle.edu.au/jon/bbp-bluegene.pdf

2010: Communication is not yet always perfect